

Chapter 4

Cloud Formation, Evolution and Destruction

We now begin to trace the journey towards a star. How long does this take? The answer is surprisingly short: a good many clouds already contain new stars and these stars tend to be young. The typical cloud cannot spend long, if any time at all, in a dormant state.

The first challenge is to accumulate gas into a molecular cloud. One dilemma is to decide what occurred first: the production of the cloud or the molecules? We all agree that a star-forming cloud must become gravitationally bound. Here, we shall investigate what this implies and what bound states are possible. To do this, we need to bring in concepts from hydrostatics and hydrodynamics. Changes in pressure and gradients in pressure take on great significance. Where will the environment's pressure confine a cloud? And where will a cloud lose pressure, deflating like a balloon? Or, where will the cloud's own gravity provide the confinement?

Theory is vital in this chapter since the changes in clouds cannot be observed on human time scales: the evolution is too slow and must be hypothesised. Many scenarios begin from a static equilibrium. This provides the initial conditions to which each protostars can be traced back. Yet the observed motions are impressively fast and we find that classical theory cannot keep up. We are led towards something more violent.

4.1 The Ages of Clouds

Age estimates for a typical Giant Molecular Cloud are discordant, lying in the range between 10^9 years and 4×10^6 years. Surely we can be a bit more precise? The higher value is derived on global grounds, simply from the total mass available, $3 \times 10^9 M_{\odot}$, and a star formation rate in the Galaxy estimated to be $3 M_{\odot} \text{ yr}^{-1}$. Accordingly, clouds were thought to be senile

and in need of support.

Secondly, star formation takes place primarily in the spiral arms where molecular clouds are either swept together or atomic clouds are compressed into clouds. Spiral arms are caused by density waves: the effects of gravity in a differentially rotating disk of material. The spread of clouds in the arms would indicate an age of order 10^8 years. This argument is also quite weak since it is not clear that clouds form exclusively in the arms. However, in most galaxies with distinct spiral arms, molecular clouds are generally confined to the arms.

A third argument employs the time scale for the production of OB stars. These are the massive stars such as formed in the Orion complex (see §3.3.1). Their radiation and winds should be capable of ripping apart a cloud within about 3×10^7 years. Even more constraining, in those regions with stars older than about 5×10^6 years there is no associated molecular gas.

Finally, the minimum time is clearly the free-fall time. This is the average time taken under gravitational acceleration for a part of a cloud to collapse unimpeded under its own gravitational pull starting from rest (by unimpeded it is meant that the cloud is devoid of all pressure support). Undertaking the calculation for a uniform cloud, we find that the total collapse time is independent of where the collapse started from and the cloud remains uniform. This yields the time

$$t_{ff} = \sqrt{\frac{3\pi}{32G\rho}}, \quad (4.1)$$

at which the density reaches infinity. This implies the collapse would last just $t_{ff} = 44 \times 10^6 / n^{0.5}$ years $= 4.4 \times 10^6$ years for an average GMC density of 100 cm^{-3} (see Eq. 3.20). One can also define a dynamical time scale R_c/σ in terms of the dispersion in velocities

$$t_{dyn} = 0.98 \times 10^6 \left[\frac{R_c}{1\text{pc}} \right] \left[\frac{\sigma_v}{1 \text{ km s}^{-1}} \right]^{-1} \text{ yr.} \quad (4.2)$$

This suggests that the observed velocities should result in a rapidly changing cloud morphology, whatever the density.

Current evidence is coming down on the side of the shorter time scales and this is changing our view on how clouds evolve. Instead of being slowly collapsing or supported objects, molecular clouds are dynamic and ephemeral, resembling terrestrial clouds. The different layers of evidence

converge towards a single conclusion: the typical cloud lifetime is just a few free-fall or dynamical time scales.

For many years, astronomers attempted to find means of support for the proposed senile cloud, in order to impede their immediate gravitational collapse. The obvious candidate was actually observed: turbulence. The width of observed emission lines implies turbulent motions within clouds. Hence, it seemed obvious to some that turbulence hinders cloud collapse and is the reason why clouds still exist.

It became increasingly evident, however, that we were struggling with this explanation. First, the origin of the turbulence was hard to pinpoint. Second, the turbulence needs continual replacement since it dissipates as fast as it is produced. Resourceful astronomers invoked particularly resilient waves called Alfvén waves (§7.2) in which energy can be temporarily stored as magnetic energy, to be re-released back into the turbulent motions. Computer simulations finally laid serious doubt, not on the existence of such waves, but on their durability.

At the same time our knowledge of clouds and the distributions of young stars had been accumulating. The hard reality was that clouds are associated with quite young stars, typically under 10×10^6 years old. This was found to hold for both the subgroups of OB stars as well as for lower mass T Tauri stars associated with dark clouds. Furthermore, very few GMCs and a minority of dark clouds are not associated with young stars. Hence the total life of clouds cannot far exceed 20×10^6 years on average. In addition the isolated dark clouds, the Bok globules, have been estimated to be a maximum of 3×10^6 years old.

The phases of rapid evolution deduced from GMC observations are sketched in Fig. 4.1. At first, only low-mass stars form in a cold dense cloud. The ultraviolet photons do not create large ionised regions and molecular gas survives. At some point of the cloud's history, high-mass stars form. What triggers this change is still not clear. An external factor is probably responsible: distant high-mass stars may be triggering larger fragments of the cloud to collapse. Or, a number of low-mass protostars might coalesce. The newly formed massive stars ionise and dissipate the local cloud, bringing star formation to an end except in more distant suburbs. Therefore, while the local cloud is dispersed, the activity of the massive stars triggers a collapse in distant waiting clouds.

There is also anatomical evidence for rapid cloud evolution: many clouds are chemically immature. Various trace molecules appear not to have had time to reach their equilibrium abundances. Instead, the regions are still

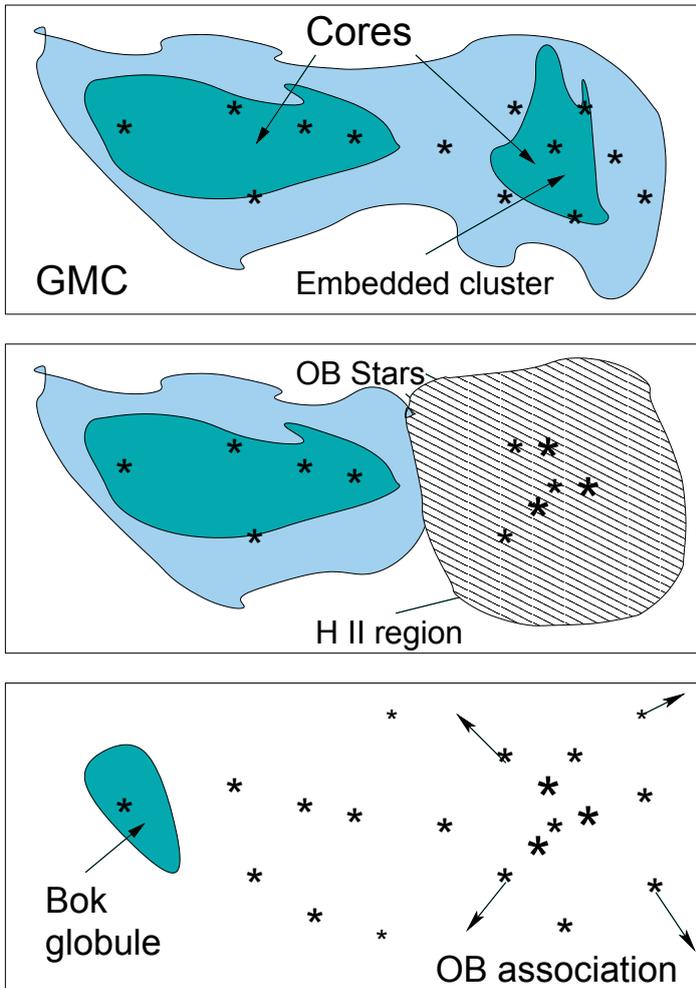


Fig. 4.1 A schematic sequence of events leading to the formation of an OB association and an isolated Bok globule out of a Giant Molecular Cloud. The evolution occurs from top to bottom in 10 Myr. Most of the cloud disperses and the final cluster is not gravitationally bound.

in the course of processing the molecules, suggesting the clouds have only existed for a few million years (see §3.3.4). It has also been found surprising that more molecules within dense cold clouds have not condensed or ‘frozen out’ onto dust grains as predicted if the clouds were stable static entities.

These enigmas have actually been around for decades. Yet the consensus was that clouds were old, forcing alternative explanations to be sought. For example, the turbulent motions could cycle the material between the interior and the surface where the exposure to ultraviolet radiation could rejuvenate the chemical state.

How do we reconcile the rapid cloud evolution with the relatively low rate of star formation, $3 M_{\odot} \text{ yr}^{-1}$, in our Galaxy? Given the molecular mass and the short lifetime, then $150 M_{\odot} \text{ yr}^{-1}$ is assembled into clouds. Accordingly, we argue that only 2% of the total cloud material gets converted into stars. The remaining 98% is dispersed!

We define the *efficiency* as the portion of a cloud which is turned into stars before the cloud disperses. A low efficiency means that the cluster of stars which form is unlikely to have sufficient mass to be bound (depending on how tightly bound the cloud is and the relative sizes of the cloud and cluster). The *global* star formation efficiency might be indeed be low but the *local* efficiency of individual clouds is estimated to be extremely high. In cloud complexes such as Orion, star formation is confined to specific small locations. Typically, 10–40% of a cloud core is converted into a cluster of stars. Such high efficiencies mean that little residual gas is left to be dispersed and a gravitationally bound cluster is likely.

We may conclude that there is overwhelming evidence in favour of transient clouds. Some fraction of a cloud collapses while most of a cloud is recycled. Clouds are dynamic: a quasi-equilibrium configuration in which gravity is held up by turbulence is not necessary.

4.2 The Origins of Clouds

4.2.1 Formation of the giant clouds

There are two types and two states of clouds. Clouds can be either diffuse or self-gravitating, atomic or molecular. These are independent conditions: even diffuse clouds are large enough for their interiors to become protected from UV light, allowing molecules to form. High Latitude Clouds (HLCs) are good examples. HLCs are molecular, and, as their name suggests, not found in the Galactic plane. They possess high turbulent pressure but are of too low a mass to be self-gravitating.

Hence, four possible cloud conditions exist. Which one occurs will depend on the site conditions, including the local UV radiation flux and the dust abundance.

We still have to determine how and where the atoms combine into molecules. This may proceed within the diffuse clouds which are subsequently compressed into observable structures. Or, diffuse atomic clouds may first be compressed into dense clouds where molecules can form efficiently.

Mechanisms which have received support for directly forming molecular clouds are as follows. In practice, each mechanism may instigate one or more of the other mechanisms and so act indirectly.

1. Agglomeration. Clouds grow by random collisions and coalescence of diffuse molecular clouds (pre-formed from atomic clouds). The process is enhanced during the passage of a spiral arms where orbits are focussed. This is the ‘bottom-up’ approach.

2. Gravitational instability. Large objects form first and fragment (a ‘top-down scenario’).

3. Collision within high-speed turbulent flows. Molecular clouds are the tips of the icebergs in the general turbulent flow: molecules form in regions of elevated density, especially after the collision of high density regions.

4. Shock accumulation. Mass is swept up into thick *supershells* by winds and supernova. The supershells eventually become gravitationally bound and fragment.

5. Streaming along magnetic fields. Large-scale fields bend and twist out and through the galactic plane since they are buoyant. Material pours down the field to magnetic ‘valleys’, locations of low gravitational potential.

6. Thermal instability. Regions of atomic diffuse gas which are slightly cooler but denser than their surroundings may cool faster, losing thermal support. Becoming even denser and cooler, radiation cooling rates may further increase, leading to a runaway process in which flattened clouds form.

The collision or random coagulation hypothesis has been around for a long time in diverse forms. It is now attacked on several grounds. First, the process is slow, taking over 100 Myr to construct a GMC. Second, there is no guarantee that a collision leads to a merger rather than to destruction. Thirdly, the clumps and clouds that we once perceived as distinct ballistic objects turn out to be part of an interconnected network.

Most conclusively, collision models are inconsistent with the observations: there is an insufficient number of small molecular clouds in existence to be able to form the GMCs. Most of the molecular mass is already in the GMCs (§3.4). It may be possible that a vast sea of small molecular

clouds has been overlooked. They could be too cold to be detected in the CO tracer. Alternatively, they may be in the diffuse form termed ‘Chaff’, the equivalent of High Latitude Clouds but in the Galactic plane. The total mass in Chaff in our Galaxy is estimated to be less than 5% of the that in GMCs (§3.3.5). That implies that to be responsible for GMCs, the molecular gas must spend only 10% of its time in the form of Chaff. This is untenable given the already short GMC lifetimes.

Hence it appears that the GMCs must have formed by condensation from atomic material in a ‘top-down’ picture. This has considerable observational support. First, dense neutral atomic envelopes to molecular clouds are quite common. In addition, some clouds in the spiral arms contain over $10^7 M_{\odot}$ of neutral atomic hydrogen. These so-called ‘superclouds’ often display sub-structure recognisable as GMCs. The smaller atomic envelopes could be in transit in either direction: dissociated molecular gas or atomic gas in the process of cooling and condensing onto the molecular cloud. It should be remarked that atmospheric clouds form similarly with water droplets condensing in certain regions where air rises, rather than being gathered together after droplet formation.

In contrast, near our Galactic centre, the molecular gas is dominant. This is not surprising since the site conditions are critical. The rate at which molecules form in these dense regions is much higher. Therefore, atoms may only survive for considerable periods in exposed sheets or tiny knots. On some days the cloud cover is thick.

The question as to what large-scale process gathered the material into clouds still remains open. Gravitational instability on a large scale provides a plausible explanation. Here, the interstellar gas is collected directly into large cloud complexes through the growth of sheared density perturbations. Density waves, which clearly collect up the gas and amplify star formation, lead to spiral arms and bars in galaxies. Gravitational instability, or ‘swing amplification’, may more generally lead to star formation in complex environments.

In a medium as turbulent as the ISM, however, the formation of gravitationally bound clouds hardly requires an instability. The fact that gravitational instability may be present suggests that cloud formation would be enhanced. Hence, our third explanation is the most convincing, with clouds resulting from the large dense regions within a fully turbulent flow. The turbulence and density in these regions are enhanced in spiral arms. The question of responsibility is then restated: what causes or drives the turbulence? We will explore the solutions in §4.4.

Star formation in the extended solar neighbourhood is also believed to have been triggered by the accumulation of gas within a spiral arm into a supercloud, which then fragmented into molecular clouds. The process was initiated some 60 Myr ago when, it is estimated, we would have been part of the Carina arm. An OB association disrupted a parental cloud blowing a gigantic bubble, pushing away a massive ring, called Lindblad's ring. This ring fragmented about 20 Myr ago to form our local GMCs such as Orion and Perseus. Eventually, new clouds were formed in Taurus and Ophiuchus. Now only remnants of the original supercloud can be recognised.

Amongst the alternative models, also likely to occur in some locations is the fourth alternative. Compressive but non-disruptive shock waves are widespread in galaxies. The blast waves from supernovae weaken as the hot cavity they produce expands. Our observationally-derived view of the interstellar medium as 'frothy', in which supernovae and H II regions sweep up shells of gas, favours the supershell mechanism for GMC formation. Galactic shocks may have other causes related to any steep pressure gradient in the interstellar medium. These shocks undoubtedly trigger the formation of stars in existing clouds, as will be discussed, but they may also be capable of compressing gas towards the critical mass where self-gravity takes over. One can envision the driven supershells as producing second generation clouds, having a moderate gathering capacity as compared to the spiral arms. Second or third generations of stars then follow, probably also accounting for our Sun.

Models constructed upon thermal instabilities and other large-scale instabilities can be discarded since they all require some disturbance to a steady state. A steady state rarely exists in the ISM although the hot diffuse gas stands the best chance of reaching such a state. Hence small diffuse clouds could be formed in this manner. The non-linear effects associated with these processes, however, may certainly be responsible for specific cloud structure, taking over command when turbulence or gravity has initiated the cloud formation process. We discuss instabilities further in §4.3.4 below.

4.2.2 *Formation of Bok globules and dark clouds*

Can stars form in isolation, or did they all form in large associations and then drift apart to appear so scattered through the night sky? The presence of young stars in Bok globules suggested that stars could indeed form in isolation. This may, however, prove to be incorrect for the following

reason. Close examination of wide regions of sky indicates that these globules are not so distant from molecular cloud complexes. They may have originally been part of these cloud complexes but lost contact. Or, they may have been associated with dense dusty ridges or filaments of atomic gas extending away from molecular clouds. These ridges may have found themselves isolated and at arm's length from the UV radiation of OB stars. Thus, molecules may have begun to form. In turn, the molecules cooled the ridge which then gravitationally collapsed and fragmented into clumps. Interclump material with a low column density is left behind which could be promptly dispersed or ionised.

4.3 The Fate of a Cloud

4.3.1 *Gradual evolution: cloud pressure*

Which clouds will form into stars? How can we decide if a cloud will collapse or disperse? To tackle these questions, we first need a mathematical description for the motions of the gas. We describe the gas as a fluid, which means we do not concern ourselves with the motions of individual molecules but with their collective motions. This is in general fine since the molecules do indeed move together, keeping the same neighbours for considerable time. As we have seen, the cross-sectional area which a molecule effectively presents to its neighbours is $\sim 10^{-15} \text{ cm}^2$ (§2.3.2). With a thermal speed of 0.2 km s^{-1} , collisions occur every $1/(\sigma n v_t)$ seconds, or about $1000/n$ years. Since n exceeds 100 cm^{-3} , the collision time is tiny compared to the cloud evolution and dynamical time scales. Therefore, a fluid treatment is correct.

The motion of a fluid element of gas is determined by the sum of the applied forces. The obvious forces on the cloud are due to pressure differences in the gas, gravity and magnetic fields. The force on a unit volume of fluid of density ρ is written in the form of the fundamental hydrodynamic equation of motion:

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla P_t + \rho \mathbf{g} + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B} \quad (4.3)$$

where $d\mathbf{v}/dt$ is the acceleration resulting from the gradient in pressure, ∇P_t , the total gravitational acceleration on unit mass, \mathbf{g} and the magnetic field B-term, called the Lorentz force. We postpone discussion of the Lorentz force until §6.6. This equation without modifying forces is often

referred to as Euler's equation. To fully describe the flow, we require three further equations: an equation expressing mass conservation, an equation expressing the energy state and an equation for the gravitational field.

In physics, the moment of inertia, I , is the name given to rotational inertia: the resistance of an object to being spun. It is the total of the contributions of all mass elements multiplied by the square of their distances from a fixed point. Therefore, I decreases if an object contracts uniformly. Conveniently, the evolution of the moment of inertia of a cloud can be considered without any further equations. To do this, the fluid equation requires some manipulation. In brief, we take components in a specific radial direction, multiply by the radial distance and integrate over the entire mass. This transforms the acceleration into the acceleration of the moment of inertia:

$$\frac{1}{2} \frac{d^2 I}{dt^2} = 2(\mathcal{T} - \mathcal{T}_s) + \mathcal{M} + \mathcal{W}, \quad (4.4)$$

where for the hydrostatic configuration, we balance three quantities:

- (1) twice the kinetic energy $2(\mathcal{T}$ which is in excess of the background cloud energy \mathcal{T}_s),
- (2) the gravitational energy, \mathcal{W} , and
- (3) the net magnetic energy, \mathcal{M} . This is a form of the virial theorem. The inertia term reflects changes in the size and shape of the cloud. For fast cloud evolution, this term is as large as the other terms. In the slow cloud evolution discussed here first, we ignore this term.

4.3.2 Long-lived isolated clouds

Now we can explore the properties of a cloud in a steady or slowly-evolving state. We can also investigate the conditions which will permit the moment of inertia to oscillate around some constant value. We would like to know what observable parameters to measure to identify a fixed cloud.

The general answer is that to maintain a cloud in equilibrium we require the pressures to balance. To show this we write all the terms in Eq. 4.4 in the form of pressures. A pressure is a force per unit area and we normally consider it to be exerted upon an area (such as the soles of our feet). Here, however, we shall also define equivalent gravitational and turbulent pressures. The point is that it takes a specific amount of energy in a unit volume of fluid to produce a pressure on the surface. In this sense, the two are equivalent. First, the thermal pressure is proportional to the number

of particles and their temperature, $P_t = n_p kT$, where k is the Boltzmann constant. We take $n_p = n(H_2) + n(He) = 0.6n$, constituting $0.5n$ from the molecular hydrogen gas and $0.1n$ contributed from helium atoms. We also average over the entire cloud and take a symbol $\hat{}$ to denote a cloud average. The pressure is then written as

$$\hat{P}_t = \hat{\rho} v_{th}^2, \quad (4.5)$$

where v_{th} is given by Eq. 3.9. The symbol $\hat{}$ denotes a cloud average.

We can also include pressure support from small-scale swirling motions by taking a *kinetic pressure* as the sum of thermal and turbulent pressures. Together they produce our observable 1D velocity dispersion σ through the relation

$$\hat{P} = \hat{\rho} \sigma^2 \quad (4.6)$$

giving a total kinetic energy of $\mathcal{T} = \frac{3}{2} \hat{P} V_c$ where V_c is the cloud volume.

The total gravitational energy depends on the detailed cloud structure. We usually assume it corresponds closely to that of a uniform sphere which takes the value

$$\mathcal{W} = -\frac{3}{5} \left(\frac{GM^2}{R} \right). \quad (4.7)$$

We now write $\mathcal{W} = -3P_G V_c$ to define an equivalent gravitational pressure P_G .

Collecting the terms together (ignoring for now the magnetic energy), we are left with a very simple equation:

$$\hat{P} = P_s + P_G. \quad (4.8)$$

That is, in virial equilibrium the cloud pressure stands in balance against the surface and gravitational pressures.

In this picture, a cloud is *gravitationally bound* if the gravitational pressure exceeds the surface pressure. Alternatively, a cloud is *pressure confined* if P_s exceeds P_G . Note, however, that if the cloud's turbulent motions were on a large scale, then there would be no external means of counteracting the accompanying pressure variations and so the cloud would disperse (unless the turbulence decays quicker).

We can now evaluate the terms. The surface pressure on a GMC is the interstellar pressure. In the classical picture of the ISM, there is a pressure balance between the three atomic components. In CGS units of pressure, we require about 4×10^{-12} dyne cm^{-2} to support the weight of the hot

galactic corona above the Galactic plane. This, in part, consists of cosmic rays and magnetic pressure, which also pervade the GMC. This probably leaves about 2.5×10^{-12} dyne cm^{-2} as applied surface pressure applied to a GMC.

Evaluating the gravitational pressure for a spherical cloud from Eq. 4.7, we obtain a dependence on just the average column density,

$$P_G = 2.0 \times 10^{-11} \left(\frac{N}{10^{22} \text{ cm}^{-2}} \right)^2 \text{ dyne cm}^{-2}. \quad (4.9)$$

The column is related how dark the cloud is through Eq. 2.5 and is given by Eq. 3.12. Therefore, the gravitational pressure is significantly higher than the surface pressure for the given typical extinction column. Consequently, *if* GMCs are in equilibrium, they are gravitationally bound rather than pressure confined, within this treatment as classical stable entities.

The easiest way to test a cloud for binding or confinement is to use the *virial parameter* α , 3.14, which is more or less equal to the critical pressure ratio \hat{P}/P_G . As discussed in §3.4, most clumps are found to be unbound but some, perhaps containing most of the mass, are potentially bound objects. The exact value for which clouds are bound, however, is more theoretical than practical, given the distorted cloud shapes and non-uniform structures within GMCs.

4.3.3 Bound globules

We now show that, even if pressure *equilibrium* is established, only certain clouds will be *stable*. The properties of clumps and isolated globules can be measured with less confusion surrounding their shape and size. We can use the virial theorem to predict the structure of self-gravitating spherical clumps which are isothermal. Although still rather ideal, such an analysis provides a first comparison. As it turns out, the comparison can be surprisingly good.

For a cloud to be in equilibrium we require a surface pressure

$$P_s = \frac{3M\sigma^2}{4\pi R^3} - \frac{3aGM^2}{20\pi R^4}, \quad (4.10)$$

where we have substituted values into Eq. 4.8 from Eqs 3.14 and 4.6. Here a is a constant close to unity, inserted to account for departures from uniformity. Isothermal is here taken broadly to mean a constant velocity dispersion σ .

We deduce that kinetic pressure is dominant if R is large but that gravitational pressure is critical if R is small, for a cloud of fixed mass. We now make the following experiment, being guided by Eq. 4.10. Suppose that the external pressure, P_s , increases. The cloud can respond in two ways. If the radius is large then the term for kinetic pressure dominates. The pressure increase means the cloud shrinks. On the other hand, if the cloud is small, then the external pressure change would be met by a change in the gravitational pressure. However, that implies the opposite reaction: the radius *increases*. This is physically impossible since there has been an increase in the pressure acting across the cloud surface. To match this, the cloud density must rise and so the cloud must contract. Consequently, if we begin with a large cloud and slowly increase the external pressure, the cloud remains stable as it contracts until a particular radius is reached. A further increase in pressure then triggers the instability: the cloud collapses.

For a fixed external pressure, we obtain the maximum mass for an isothermal sphere to be stable from Eq. 4.10:

$$M_{BE} = 1.18 \frac{\sigma^4}{(G^{3/2} P_s)^{1/2}}, \quad (4.11)$$

which is called the Bonner-Ebert mass (the appropriate value of ‘ a ’, involves solving the hydrostatic form of Eq. 4.3 and Eq. 4.14 below). With this mass, a stable sphere has its greatest central concentration. The core density is 13.98 times higher than the density just inside the surface. In terms of solar masses, this mass is quite low:

$$M_{BE} = 0.96 \left(\frac{T}{10 \text{ K}} \right)^2 \left(\frac{P_s}{2 \cdot 10^{-11} \text{ dyne cm}^{-2}} \right)^{-1/2} M_{\odot}, \quad (4.12)$$

which indicates that cold cores of high mass (e.g. $> 5 M_{\odot}$) are unlikely to be supportable by thermal pressure alone. The typical high surface pressure *inside* a GMC has been taken. Corresponding to the mass is the critical radius for marginal stability,

$$R_{crit} = 0.41 \frac{G M_{BE}}{\sigma^2}. \quad (4.13)$$

Clouds larger than R_{crit} are stable.

These values provide initial conditions for some theories of cloud evolution which begin from hydrostatic equilibrium. How the collapse can proceed will be discussed in §6.3.

4.3.4 The Jeans mass

Theoretically, stars can condense out of an almost perfectly uniform gas. Even where turbulent motions are negligible, clouds could still form. This requires an instability to be present. One instability is caused by a breakdown in the delicate balance between heating and cooling which can occur under some conditions. When a gas cloud is thermally unstable, a local small perturbation will cool slightly faster than its surroundings. Its drop in pressure causes it to collapse. The fact that the density increases usually causes it to cool even faster. The collapse thus accelerates and a tiny perturbation grows into a discrete cloud.

Our interest is in obtaining clouds which can gravitationally collapse into a protostar. This can be achieved directly through the Jeans instability: disturbances to regions containing sufficient mass will separate out from the uniform background.

To show this, we need to delve into a ‘stability analysis’. We eliminate the gravity term \mathbf{g} in Eq. 4.3 by substituting the density field ρ which produces it. The gravitational influence of all the material on each fluid particle is much more conveniently written as Poisson’s equation,

$$\nabla \cdot \mathbf{g} = -4\pi G\rho. \quad (4.14)$$

If we inspect this, we notice that a uniform medium actually generates gravitational acceleration in a particular direction! This abnormality is the ‘Jeans swindle’, invoked by the assumed design of a truly uniform and static medium filling our entire Universe. We search for solutions to Eq. 4.3 involving small sinusoidal oscillations of frequency ω and wavelength λ . Fortunately, the cheat dissolves when we consider these first order effects. The result is a form of a *dispersion relation*:

$$\omega^2 = c_s^2 \left(\frac{2\pi}{\lambda} \right)^2 - 4\pi G\rho. \quad (4.15)$$

where c_s is the sound speed, the speed of waves in the absence of gravity. Ignoring the final term, we have the usual relation for a sound wave, which is non-dispersive (substituting $c_s \sim 331 \text{ m s}^{-1}$ and middle C of $\omega = 262 \text{ Hz}$ yields \sim one metre waves). Hence, this equation demonstrates the propagation of acoustic waves when modified by self-gravity. The two terms on the right represent ‘restoring forces’ which attempt to re-expand

compressed regions. For this reason, for long wavelengths,

$$\lambda > \lambda_J = \sqrt{\frac{\pi c_s^2}{G\rho}}, \quad (4.16)$$

gravity overcomes the acoustic restoring force. This is called the Jeans criterion for gravitational instability, after the work of Sir James Jeans, published in the early 20th century. Taking a spherical disturbance this directly translates into a minimum mass

$$M_J = \frac{4\pi}{3} \left(\frac{\lambda_J}{2}\right)^3 \rho = \frac{\pi}{6} \left(\frac{\pi}{G}\right)^{3/2} c_s^3 \rho^{-1/2} \quad (4.17)$$

or

$$M_J = 2.43 \left(\frac{T}{10 \text{ K}}\right)^2 \left(\frac{P}{2 \times 10^{-11} \text{ dyne cm}^{-2}}\right)^{-1/2} M_\odot. \quad (4.18)$$

The Jeans mass exceeds that contained within Bonnor-Ebert spheres with the same surface conditions. This reflects the fact that a Bonnor-Ebert sphere contains matter of exclusively higher density than the surrounding medium whereas only some of the material within a Jeans length will collapse. In terms of the hydrogen number density, n , the Jeans mass is

$$M_J = M_1 \left(\frac{T}{10 \text{ K}}\right)^{3/2} \left(\frac{n}{10^5 \text{ cm}^{-3}}\right)^{-1/2} \quad (4.19)$$

for a molecular gas, where $M_1 = 1.18 M_\odot$ under the above assumptions but takes a value $M_1 = 0.78 M_\odot$ under a full spherical analysis. Sometimes, the Jeans mass is defined as $\rho\lambda_J^3$ and then $M_1 = 2.3\text{--}3.4 M_\odot$, depending on the particular assumptions.

According to a classical picture originally proposed by Fred Hoyle, a cloud or a part of a cloud with mass above the Jeans mass collapses under gravity. The Jeans mass falls as the density increases provided the collapse remains isothermal. Therefore, each part will also fragment and fragment again. A hierarchy of cloud sizes is systematically created until the collapse ceases to be isothermal. At the point when the gravitational energy, released through compressive heating, cannot escape the system in the form of radiation, the temperature of the fragments rises. According to Eq. 4.19, the Jeans mass now also rises and so the fragmentation stops. This is called ‘opacity-limited fragmentation’ since the heat is trapped when the core becomes opaque.

One observed inconsistency with this scenario is that the minimum Jeans mass, given the opacity due to dust, is only about 7 Jupiter masses ($0.007 M_{\odot}$), much smaller than the observed characteristic stellar mass near $1 M_{\odot}$ (see §12.7). Furthermore, simulations show that the efficiency of the process is drastically reduced through the overall collapse of the embedding medium and non-linear damping effects as the speed of collapse becomes high. Far more probable is that bound cores develop out of the large range of transient structure in the cloud produced by strong turbulence. Nevertheless, the Jeans criterion is still a central feature in all the theories, providing the critical mass which must accumulate to initiate gravitational collapse.

4.4 Summary

In the classical theory for star formation we first searched for equilibrium between self-gravity and pressure gradients. Astrophysicists took into account a contribution from turbulence in the form of a turbulent pressure which acts in the same way as thermal pressure. That is, as a microphysical concept rather than a fluid dynamics process. An effective sound speed then replaces the true sound speed. Such configurations may serve as initial conditions from which we can simulate the collapse all the way to a star.

Do we find that clouds evolve towards static states in order to provide the proposed initial conditions? In general, approximate virial equilibrium has been confirmed in the GMCs as a whole. But, as we have emphasised, the GMCs are non-equilibrium distorted structures which evolve rapidly.

On the other hand, quiescent molecular clumps and cores do exist which appear to roughly match the static solutions discussed above. The dilemma remains of whether these could be required states or just exceptional states produced by chance in dynamic accidents.