Homework 6

Due: Wednesday June 1 2005, only the first 3 problems to be handed in.

No late homework. Always justify your answers and show your work.

1. Suppose the difference in rest energy of the neutron and proton were $Q_n = (m_n - m_p)c^2 = 0.129$ MeV, instead of 1.29 MeV, with all other physical parameters unchanged. Estimate Y_{max}, the maximum possible mass fraction in ^{4}He, assuming that all available neutrons are incorporated into ^{4}He nuclei.

2. A fascinating bit of cosmological history is that of George Gamow's prediction of the Cosmic Microwave Background in 1948 (unfortunately, his prediction was premature; by the time the CMB was actually discovered in the 1960's, his prediction had fallen into obscurity). This problem reproduces Gamow's line of argument.

   Gamow knew that nucleosynthesis must have taken place at a temperature $T_{nuc} \simeq 10^9$ K. He also knew that the universe must currently be $t_0 \sim 10^{10}$ years old. He then assumed that the universe was *flat and radiation dominated*, even at the present time.

   (a) With these assumptions, what was the energy density of the universe ϵ at the time of nucleosynthesis, when $T_{nuc} = 10^9$ K?

   (b) What was the Hubble parameter H at the time of nucleosynthesis?

   (c) What was the age of the universe when $T_{nuc} = 10^9$ K?

   (d) Given the present age, what should the present temperature of the Cosmic Microwave Background, T_0, be?

   (e) If we then assume that the universe changed from being radiation dominated to matter dominated at a redshift $z_{rm} > 0$, will this increase or decrease the temperature T_0, for fixed values of T_{nuc} and t_0?

3. Say whether these statements are true or false, explaining why briefly.

   (a) The reason why heavy elements are synthesized in stars and not in the Big Bang is because stars live longer than the age of the universe when the temperature was right for making nuclei, and they are denser than the matter in the universe was at this time.

   (b) For the Benchmark model, which includes a cosmological constant, the age of the universe at redshift $z = 10$ was shorter than in a flat space model containing only matter, with the same present Hubble constant.

   (c) The Sachs-Wolfe effect is the process that generates Cosmic Microwave Background fluctuations from the Doppler effect due to the motion of the baryon-photon fluid in the last scattering surface.

(d) More than half of all the helium that exists today in the Earth was created in nuclear reactions that occurred within the first five minutes after the Big Bang.

4. For blackbody radiation, the energy density per unit frequency is given by

$$\epsilon_\nu \, d\nu = \frac{8\pi h \nu^3 d\nu}{c^3 \left[\exp(h\nu/kT) - 1\right]} \ . \tag{1}$$

Since the energy of one photon is $h\nu$, the number density of photons is given by the same expression above divided by $h\nu$.

(a) Calculate the present density of photons in the universe, knowing that the Cosmic Microwave Background temperature is $T_0 = 2.725$.
Note: you will find it useful to know that

$$\int \frac{x^2 \, dx}{e^x - 1} \simeq 2.404 \ . \tag{2}$$

(b) If the deuterium measurements require a baryon to photon ratio of $\eta = 5.5 \times 10^{-10}$, what must the current density of baryons be?

(c) Assuming that the Hubble constant is $H_0 = 70 \, \mathrm{km\,s^{-1}\,Mpc^{-1}}$, calculate what Ω_b is (where the subscript b stands for baryons).

5. Assume that our universe is described by the Benchmark model, with flat space and a cosmological constant $\Omega_{\Lambda 0} = 0.73$, matter density $\Omega_{m0} = 0.27$, and Hubble constant $H_0 = 70 \, \mathrm{km\,s^{-1}\,Mpc^{-1}}$ at the present time t_0.

(a) At a time $t \gg t_0$, what will be the value of the Hubble constant?

(b) Suppose that the vacuum energy of the cosmological constant decays instantaneously into radiation at cosmic time $t = 50t_0$. What will be the energy density of these photons immediately after the decay?

(c) What will be the energy density of matter at $t = 50t_0$?

(d) As the universe continues to expand after $t = 50t_0$, radiation will start decreasing its energy density faster than matter. At what future time will the universe again be dominated by matter?

6. Suppose that the luminosity of the Milky Way galaxy has been constant at $2 \times 10^{10} L_\odot$ for the past 10^{10} years. Most of the energy emitted has been powered by the fusion of hydrogen into helium. How many helium nuclei have been created within stars in our Galaxy over its lifetime? If the baryonic mass in our Galaxy is $10^{11} M_\odot$, by what amount has the helium fraction Y of our galaxy been increased over its primordial value $Y = 0.24$?

(Note: the increase in helium you will calculate here is not the same as the increase in the helium content that can be measured from the chemical abundance of the interstellar medium, or of the stellar surfaces which reflect the medium from which they formed. The latter quantities depends on the helium that is ejected from stars over their lifetime, and there is a substantial amount of helium and heavier elements formed within stars that remains forever locked up in stellar remnants such as white dwarfs and neutron stars).