Homework 5

Due: Monday May 23 2005 at 1:30pm

No late homework. Always justify your answers and show your work.

1. The Sun is moving around the center of the Milky Way galaxy along a roughly circular orbit at radius \( R = 8 \) kpc, with a velocity \( v = 210 \) km s\(^{-1}\). Let us approximate the mass distribution and gravitational potential of the galaxy as spherically symmetric.

   (a) What is the total mass inside \( R \)?

   (b) If the mass density varies with radius as \( \rho \propto r^{-2} \), then what is the density at radius \( R \)? Express it in units of proton masses per cm\(^3\) (the proton mass is \( m_p = 1.672 \times 10^{-24} \) g).

   (c) For the benchmark cosmological model, with \( \Omega_{\Lambda} = 0.73 \) and \( H_0 = 70 \) km s\(^{-1}\) Mpc\(^{-1}\), what is the density of the cosmological constant (or dark energy accounting for it)? Express it in the same units as in the previous question.

   (d) The dark energy is spread out uniformly in the universe and it causes a gravitational repulsion which accelerates the expansion of the universe. Do you think the cosmological constant may be strongly affecting the dynamics of stars in our galaxy?

2. Imagine a class of astronomical objects that are both standard candles and standard yardsticks. In other words, we know both their luminosities \( L \) and their physical sizes \( \ell \). The surface brightness of an object, \( \Sigma \), is its observed flux divided by its angular area, or solid angle on the sky: \( \Sigma \propto f/\theta^2 \), where \( f \) is the flux and \( \theta \) the angular size. How does the surface brightness depend on redshift for a general cosmological model, for these objects with fixed \( L \) and \( \ell \)?

3. We know from observations that the intergalactic medium is currently ionized. Thus, at some time between the epoch of recombination at \( z_{\text{rec}} \approx 1100 \) and the present time, the intergalactic medium must have been reionized. From observations of the microwave background brightness fluctuations and polarization fluctuations, we can measure the optical depth to electron scattering of the microwave background radiation by the ionized intergalactic material. Assume that the baryonic medium instantaneously became completely reionized at some time \( t_* \), corresponding to some redshift \( z_* \). If the optical depth is 0.12, compute the value of \( z_* \) that gives this optical depth for the benchmark model (flat space with \( \Omega_{m0} = 0.27 \)).

4. The following problem is not to be handed in as homework, but is for additional practice only (a solution will be given later).

   Suppose that you are in a spatially flat universe containing a single component with a unique equation-of-state parameter \( w \); \( p = w \epsilon \).
(a) What is the comoving distance to an object at redshift $z$?
(b) What is the angular diameter distance to an object at redshift $z$?
(c) What is the luminosity distance to an object at redshift $z$?
(d) At what redshift will the angular diameter distance have a maximum value?
(e) What will this maximum value be?