Homework 3

Due: Monday 4/25/2005 at class time

No late homework. Always justify your answers and show your work.

1. A civilization that wants to conquer the universe, which is homogeneous and isotropic, and hence is described by the Friedmann-Robertson-Walker metric, is getting ready to send out soldiers in all directions to invade all the universe out to a proper distance \( d_p \). Every soldier leaves the galaxy where the civilization was born, and travels through the universe with its spaceship along a geodesic, out to a distance \( d_p \) from the original galaxy. At the end of the invasion, which occurs at a fixed cosmic time \( t \), all the soldiers stand on a spherical surface at a proper distance \( d_p \) from their original galaxy. The total volume that has been invaded is the volume inside this spherical surface.

What is the total volume invaded? Answer this question for the following three cases:

(a) A flat metric \( (k = 0) \).
(b) A closed metric \( (k = +1) \) with radius of curvature \( R \) at the cosmic time \( t \) when the invaded volume and the proper distance \( d_p \) is measured.
(c) An open metric \( (k = -1) \) with radius of curvature \( R \) at the cosmic time \( t \) when the invaded volume and the proper distance \( d_p \) is measured.

2. For each one of the following statements, say whether they are true or false, and write a brief explanation (usually only one sentence) that says why it is true or false.

(a) The redshift of a galaxy is given by \( z = H_0 r / c \), where \( r \) is its distance, as long as the redshift is small compared to one and peculiar velocities (departures from the Hubble flow) are negligible.

(b) If a traveler leaves the Earth at half the speed of light, reaches out to a distance of half a light-year during a year of travel, then turns around instantaneously and comes back to the Earth at the same constant speed (arriving 2 years after the departure according to the people in the Earth), the watch of the traveler will show that the trip has taken only a time \( (2/\sqrt{3}) \) years.

(c) If the universe is open, triangles in space at a fixed cosmic time could have the sum of their three angles be either greater or smaller than \( \pi \).

(d) If the redshift of a galaxy is 2, it means that at the cosmic time when the light we see now was emitted, the proper distance to this galaxy was three times smaller than the present one.

3. Define cosmic time.