

Midterm Exam, May 2 2005

1. For each one of the following statements, say whether they are true or false, and write a brief explanation (usually only one sentence) that says why it is true or false.
 - (a) If a galaxy that has no peculiar velocity emits a photon at wavelength λ_e at the epoch corresponding to redshift z , and we observe the photon today at wavelength λ_{obs} , it is always exactly true that $\lambda_{obs} - \lambda_e = z\lambda_e$, no matter how large z may be and no matter what cosmological model we are considering.
 - (b) In Newton's theory of gravity, one can understand the evolution of the scale factor of the universe by considering a finite sphere cut out from the universe, and one can already see in this theory that there are models in which the expansion of the universe can accelerate.
 - (c) For all cosmological models, the present age of the universe is equal to $1/H_0$, where H_0 is the present Hubble constant.
 - (d) A trip from the Solar System to the center of the galaxy (25000 light-years away from the Sun) could in principle take only 25 years according to the watch of the traveller, if the traveller were moving as fast as 0.999 times the speed of light.
 - (e) The equivalence principle says that no experiment can distinguish a frame of reference that is in free-fall from a frame of reference in flat space-time (that is to say, with no gravity) that does not accelerate in any way, as long as tidal accelerations in the free-falling frame are not important.
 - (f) The metric $ds^2 = dr^2 + r^2d\theta^2$ is that of a two-dimensional space with zero curvature; in other words, it is the metric of a flat two-dimensional space.
 - (g) If the universe is flat, two galaxies that are both at a comoving distance r from us and with an angular separation α between them on the sky (in other words, the directions from us to each galaxy subtend an angle α) are at a proper distance $2a(t)r \sin(\alpha/2)$ from each other, where $a(t)$ is the scale factor.
 - (h) If the universe is closed, two galaxies that are both at a comoving distance r from us and with an angular separation α between them on the sky are at a proper distance from each other that is less than $2a(t)r \sin(\alpha/2)$, for all cosmological models as long as the space geometry is closed.
 - (i) A galaxy seen at a redshift of 2 is observed at the cosmic time when the universe had a mean density of matter that was 27 times larger than today, in all cosmological models.
 - (j) The energy density of radiation declines more slowly than the energy density of matter as the universe expands.

2. Define geodesic.
3.
 - (a) Imagine a universe with a flat space metric that is filled only with radiation (and no matter or any other component), with the spectrum of a blackbody. At the time when the radiation has a temperature $T = 3$ K, what would be the energy density in the universe?
 - (b) At this time when $T = 3$ K, and remembering again that the space metric is assumed to be flat and that the *only* energy density is the radiation, what would be the Hubble constant? Find the answer first in s^{-1} , and then convert it to units of $\text{km s}^{-1} \text{Mpc}^{-1}$.
 - (c) What would be the age of the universe (cosmic time t) when the radiation has a temperature $T = 3$ K (remember again: it is a flat universe that contains *only* radiation)?
 - (d) What would be the age of the universe when the radiation has some other temperature T ? Write down the answer for the age t as a function of T .