Astro-682 *Spring 2005*

Final Exam, June 8 2005

1. For each one of the following statements, say whether they are true or false, and write a brief explanation (usually only one sentence) that says why it is true or false (4 points, 0.4 points each question).

   (a) For any model of the universe in which space is flat, and if r is the comoving distance to an object, the angular diameter distance is always equal to $r/(1+z)$, independently of the abundances of matter, radiation, and vacuum energy.

   (b) The linear density fluctuations in the dark matter did not start growing until after the epoch of recombination.

   (c) If the sum of the densities of all matter, radiation and vacuum energy in the universe is greater than the critical density, then (assuming the validity of General Relativity and the cosmological principle) space must have a constant positive curvature, and therefore it must have the geometry of a three-dimensional spherical surface.

   (d) In a cluster of galaxies, the velocity dispersion of the galaxies is about the same as the average velocity dispersion of the particles in the hot, X-ray emitting gas.

   (e) At the time of nucleosynthesis, when the temperature of the radiation was $\sim 3 \times 10^8$ times larger than today, the density of baryons in the universe was also 3×10^8 times larger than today.

   (f) General Relativity predicts that a ray of light passing through a cluster of galaxies, where the galaxies are moving at speeds much less than c, will be deflected by an angle equal to that predicted by Newton's theory when light is assumed to be made of particles travelling at a speed c.

   (g) If the universe were open, the first peak in the Δ_l spectrum of the Cosmic Microwave Background anisotropies would occur at smaller values of l than is observed.

   (h) The reason why primordial deuterium has a fractional abundance as low as 3×10^{-5} is that neutrons decayed too fast, and so not much deuterium could ever be made.

   (i) If the reaction

   $$p + e^- \qquad n + \nu_e \ . \tag{1}$$

   had a much larger cross section (i.e., occurred at a much larger rate), therefore allowing the proton and neutron abundances to remain in thermodynamic equilibrium until a later time, much more helium would have been made in the Big Bang.

   (j) Whereas in the Big Bang model with an initial singularity, distant regions of the universe that we observe today on the Cosmic Microwave Background were never in communication with each other in the past, in the inflationary model all such regions were previously in communication.

2. (3 points, 0.75 points each question) At the time when most of the helium was made in the universe, the temperature of the radiation was about $T = 10^9$ K.

   (a) What was the energy density of radiation in the universe at that time?

   (b) The WMAP mission has measured a present temperature of $T_0 = 2.725$ K for the cosmic microwave background, and a baryon density parameter $\Omega_{b0} = 0.045$ for $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. What was the density of baryonic matter at the time when $T = 10^9$ K?

   (c) We know that when hydrogen fuses into helium, about 0.7% of the rest-mass energy of matter is released as energy. What was the density of energy released as a result of fusing 25% of all the hydrogen in the universe into helium, at the time when $T = 10^9$ K?

   (d) Compare the total energy released by the fusion of hydrogen into helium with the energy density that was present in the radiation. Was the radiation energy strongly increased by the fusion process?

3. (3 points, 0.75 points each question) Imagine that the universe is open, with a present matter density parameter $\Omega_{m0} < 1$, no vacuum energy, and no radiation, with present Hubble constant $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

   (a) What is the present radius of curvature of the universe? (*Hint:* if you forgot the formula for the radius of curvature, go back to Friedman's equation, and apply it at the present time remembering that $\dot{a}/a = H_0$ at present and the energy density is equal to Ω_{m0} times the critical density).

   (b) Find the radius of curvature in Megaparsecs for $\Omega_{m0} = 0.3$.

   (c) What is the comoving distance (coordinate r in Robertson-Walker metric) to the horizon, at $z = \infty$, in this open model? Give your answer in terms of H_0 and Ω_{m0}. To save you some time doing integrals, you may find the following formula helpful:

$$\int_0^\infty \frac{dx}{(1+x)\sqrt{1+Ax}} = \frac{1}{\sqrt{1-A}} \log \frac{1+\sqrt{1-A}}{1-\sqrt{1-A}} \,, \tag{2}$$

   where A is a constant.

   (d) Show that in this open model, the product $S_k(z) = d_A(z) \times (1+z)$, where $d_A(z)$ is the angular diameter distance, has the limit as z goes to infinity

$$S_k(z=\infty) = \frac{2c}{H_0 \Omega_{m0}} \,. \tag{3}$$