

Modeling of solar energetic particles in interplanetary space

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Abstract. Solar energetic particles (SEPs) in the interplanetary (IP) medium are transported under the influence of electromagnetic fields of the solar wind. These fields consist of the smooth background fields, which can be modeled by the MHD equations governing the expansion of the solar wind, and of the small-scale fluctuations (waves or turbulence) that scatter the particles in pitch angle and act as agents enabling their acceleration at IP shock waves. We review theoretical models of SEP transport and acceleration in the IP medium. We start from the simple analytical approaches (diffusion models), which assume quasi-isotropic particle distributions, and then continue to the more accurate numerical approaches based on the focused transport equation, not making this simplifying assumption. A careful analysis of two SEP events, an impulsive and a gradual one, is presented and the spatial scaling of their peak intensities, differential fluences and time-integrated net fluxes is discussed. We conclude that rather simple scaling laws for these quantities can be obtained for impulsive events but no simple scaling laws can be expected to govern the gradual SEP events.

Introduction

Solar energetic particle (SEP) events are one of the main components of the solar driven **space weather**: producing most of the energetic particle **fluence** between 1–100 MeV in the **interplanetary (IP) medium**, they introduce an important radiation risk for space missions in the IP space. In ad-

dition, their effects include elevated radiation dose rates and high frequency (HF) radio blackouts at polar airline routes.

Since the 1980's **SEP events** have been divided in two classes, impulsive and gradual (Cane et al. 1986). The classification is based on bimodal distributions observed in many variables characterizing the events: **impulsive SEP events** are related to impulsive X-ray **flares**, they are typically of short duration (from hours to days) and low intensity, they are electron rich, and their ion abundance ratios show enhancements in ^3He and heavies relative to the coronal abundances (e.g., Reames 1999). **Gradual SEP events** are related to **coronal mass ejections** (CMEs) and gradual X-ray flares, their duration is longer (from days to a week), they are proton rich and their ion abundance ratios agree with those of the coronal plasma (Reames 1999). It is rather commonly accepted that impulsive events are accelerated in solar flares and gradual SEP events at coronal and IP shocks related to CMEs (Reames 1999). As the sensitivity of the SEP measurements improved as a result of the ACE, Wind, and SOHO missions in the 23rd solar cycle, it was found that the division between the two classes is not as clear as previously believed: a third class of SEP events, i.e., hybrid or mixed events, was introduced (Kocharov and Torsti 2002 and references therein) to include events that look like gradual events from the point of view of their electromagnetic associations, their duration and magnitude, but show properties of impulsive events, e.g., in their ion abundance ratios implying either a direct flare-accelerated component (Cane et al. 2003) or the shock acceleration of supra-thermal remnants in the corona from previous impulsive flares (Tylka et al. 2001).

In this paper we will consider the SEP transport in the IP space, governed by the large-scale heliospheric electromagnetic fields and the wave-particle interactions between the SEPs and the low-frequency magnetic fluctuations of the solar wind plasma. We will start by describing the relevant particle transport equations and then consider numerical modeling of SEP events paying special attention to the spatial development of space-weather relevant quantities in the inner heliosphere.

Transport equations

Diffusion models

The earliest modern modeling efforts to describe the propagation of particles in the IP medium were based on **diffusion–advection model** of

Parker (1965). This approach has still important applications in the transport of galactic and anomalous cosmic rays in the heliosphere. For SEPs, however, anisotropies and time dependence are very important factors, so only under strongly turbulent conditions can the approach yield accurate results for modeling SEP events. For more general considerations, like for modeling the spatial dependence of SEP event peak fluxes or event fluences, this approach may still give a reasonable starting point.

In the crudest approximation, one can neglect the effects of the solar wind expansion and just consider particle transport in the turbulent IP medium as **diffusion**. Assuming that the particles diffuse only along the IP magnetic field with a spatial diffusion coefficient $D = v\lambda/3$ we can model the transport using only one spatial coordinate, the radial heliocentric distance r . Here, v is the particle speed and λ is the **mean free path** of the particles parallel to the magnetic field related to the amount of fluctuations in the magnetic field, but normally regarded as a free parameter of the transport model. Thus, the **SEP transport equation** can be written as

$$\frac{\partial n_p}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 D_{rr} \frac{\partial n_p}{\partial r}, \quad (1)$$

where $n_p(r,t) = d^4N/(d^3r dp)$ is the particle density per unit momentum, $D_{rr} = D \cos^2 \psi$ is the radial diffusion coefficient and ψ is the angle between the radial direction and the local magnetic field.

Although one can justify the application of Eq. (1) to only a small fraction of SEP events, it has the attractive feature that it can be analytically solved for an impulsive injection of particles from the Sun. The result is (Wibberenz et al. 1989)

$$n_p = \frac{dN}{dp} \frac{1}{r^3} \frac{2-b}{\Gamma\{3/(2-b)\}} \left(\frac{r^2}{(2-b)^2 D_{rr} t} \right)^{3/(2-b)} \exp\left(\frac{-r^2}{(2-b)^2 D_{rr} t} \right) \quad (2)$$

where dN/dp is the momentum spectrum of particles injected to the IP medium per steradian at the solar surface and $D_{rr} \propto r^b$ with $b < 2$ has been assumed.

If the injection is extended in time, the solution of the transport equation can be obtained by using Eq. (2) as a Green's function, i.e., convolving the function with an extended injection profile $Q(E, t)$. Usually, the task of the modeler is to find out the time profile of the SEP injection as well as the IP mean free path. Since the time scales of the coronal and IP acceleration processes may be extended, a mere fit of the modeled omni-directional differential particle intensity, $I = (1/4\pi) n_p$, (hereafter, intensity) to observed intensity–time profile does not provide a unique solution to the problem.

To reduce the ambiguity between the contributions of the injection and the transport model to the result, one needs to model the anisotropies of the particle distribution as well. In the simplest approach, one considers the net flux of particles per unit momentum across a spherical surface, which in a diffusion model is given by **Fick's law**

$$S_r = -D_{rr} \frac{\partial n_p}{\partial r}. \quad (3)$$

This quantity, with a simple relation to the first-order anisotropy, has sensitivity to the value of the diffusion coefficient and, thus, helps to separate the effects of the time-extended particle injection from the effects of particle transport in the solar wind.

We can now obtain a few spatial scaling laws from the diffusion equation and its analytical solution for short-duration solar injections:

(i) the time-integrated radial net flux scales like

$$\int S_r dt \propto r^{-2};$$

(ii) the time of maximum intensity for an impulsive injection scales like

$$I_{\max}(r) \propto r^{-3};$$

(iii) the time-integrated intensity (or fluence) scales like

$$\int I(r,t) dt \propto 1/(rD_{rr}).$$

Thus, only the time-integrated radial net flux scales like $1/r^2$, although this scaling law is often used for scaling the fluences and the peak intensities as well. We must note, however, that the scaling laws obtained from the diffusion equation are not always valid, but at least the following conditions have to be met: (a) the mean free path of the SEPs is much smaller than the heliocentric distance of the observer, i.e., $\lambda_{rr} \ll r$; (b) the time of maximum intensity of the solution (2) is much smaller than the adiabatic cooling time, i.e., $r/\lambda_{rr} \ll v/(2V)$, where V is the solar wind speed; (c) the duration of the injection at the Sun is shorter than the time of maximum intensity; and (d) the site of the injection, r_0 , is close to the Sun, i.e., $r_0 \ll r$. For the validity of scaling-law (i), only the conditions (b) and (d) are necessary, the remaining scaling laws require all the conditions. For cases not meeting these (rather strict) conditions, we need to resort to numerical methods to investigate the scaling.

Focused transport model

Because of the **focusing** effect due to the outwards decreasing magnetic field magnitude in the inner heliosphere, the anisotropies become too large

for the diffusion model to be applicable if the mean free path in the IP medium is comparable to the radial distance from the Sun. We, thus, need a transport equation describing the evolution of the **particle distribution function** $f(s, p, \mu, t)$, which gives the number of particles per unit volume of the six-dimensional phase space (\mathbf{r}, \mathbf{p}) . In the focused transport approximation, the distribution function is governed by streaming along the magnetic field lines, by scattering off the fluctuations of the magnetic field and by magnetic focusing (i.e., mirroring) in the outwards decreasing magnetic field. The distribution function is a function of the coordinate measured along the mean magnetic field, s , the particle momentum p , the cosine of **pitch-angle** μ , i.e., the angle between the magnetic field and the velocity vector of the particle, and time t . Its relations to the particle density and streaming per unit momentum are

$$\begin{aligned} n_p(s, t) &= 2\pi p^2 \int f(s, p, \mu, t) d\mu \\ S_p(s, t) &= 2\pi p^2 \int v\mu f(s, p, \mu, t) d\mu \end{aligned} \quad (4)$$

where now the streaming has been defined with respect to a unit area perpendicular to the magnetic field instead of a spherical surface like in Eq. (3). The radial streaming can be obtained from this quantity by multiplying with $\cos\psi$.

The equation governing the evolution of the particle distribution is the **focused transport equation** (Roelof 1969)

$$\frac{\partial f}{\partial t} + v\mu \frac{\partial f}{\partial s} + \frac{1-\mu^2}{2L} v \frac{\partial f}{\partial \mu} = \frac{\partial}{\partial \mu} D_{\mu\mu} \frac{\partial f}{\partial \mu} \quad (5)$$

where the second term describes streaming of particles along the magnetic field lines, the third term describes the focusing of particles because of the mirror force and the last term accounts for the effect of magnetic fluctuations, which is modeled by **pitch-angle diffusion**. This simple form of the equation still neglects adiabatic deceleration (see, Ruffolo 1995, for the full equation), but at the energies of interest for space weather, this is typically a small effect. The pitch-angle diffusion coefficient has the form

$$D_{\mu\mu} = \frac{1}{2} (1-\mu^2) \varphi(\mu), \quad (6)$$

where, following the **quasi-linear theory** (Jokipii 1966; Jaekel & Schlickeiser 1992), the **scattering frequency** is usually modeled as $\varphi(\mu) = \varphi_0 |\mu|^{q-1}$, with $q \in [1, 2]$ and related to the **scattering mean free path** as

$$\lambda = \frac{3v}{4} \int \frac{1-\mu^2}{\varphi(\mu)} d\mu. \quad (7)$$

Modeling of IP particle transport and injection in SEP events

In the following, we will consider two examples of modeling SEP events in the IP medium. Similar approaches have been used to model dozens of particle events by a number of authors (e.g., Heras et al. 1992; Torsti et al. 1996; Lario et al. 1998; Aran et al. 2004). We fit the injection and transport parameters to SEP observations at 1 AU using the focused transport equation, and then study the modeled event at different distances from the Sun, paying special attention to the inner parts of the heliosphere. This region will be accessed by several spacecraft in the next decade, including ESA's [BepiColombo](#) and [Solar Orbiter](#) missions.

Impulsive SEP events

It is reasonable to consider the source of the SEPs in impulsive events to be close to the Sun. The focused transport equation is first solved for an impulsive injection from the corona to the IP magnetic field to obtain a Green's function of IP transport. The SEP injection at the root of an IP flux tube is then parameterized using a convenient mathematical function, e.g., the diffusive Reid–Axford profile (Reid 1964) with rise time and decay time constants. This is convolved with the simulated Green's function and the result is compared with observations of intensity and anisotropy. The parameters of the injection and IP transport are then varied until the best fit is found.

We have applied this method to the electron intensities measured by the EPAM instrument (Gold et al. 1998) onboard ACE during the impulsive event of 1 May 2000 (Fig. 1). The event is related to an impulsive M1 class X-ray flare from N20° W54° peaking at 10:27 UT. Although the event is associated with a narrow, fast CME as well, its characteristics are typical to an impulsive event (Kahler et al. 2001; Ho et al. 2003; Mason et al. 2004). We have fitted the four electron energy channels of the EPAM/LEFS-60 telescope (at 45–312 keV) using sectorized intensities sensitive to anisotropies as well. Using a model of the full directional response of each sector, we calculate the sectorized intensity of the electrons obtained

from a convolution of a Monte Carlo simulated Green's of IP transport and the Reid–Axford profile. The best fit is found varying the parameters trying to minimize χ^2 . Details of the modeling will be published elsewhere, but the simulation is very similar to those previously used in the studies of IP transport (e.g., Torsti et al. 1996; Kocharov et al. 1998).

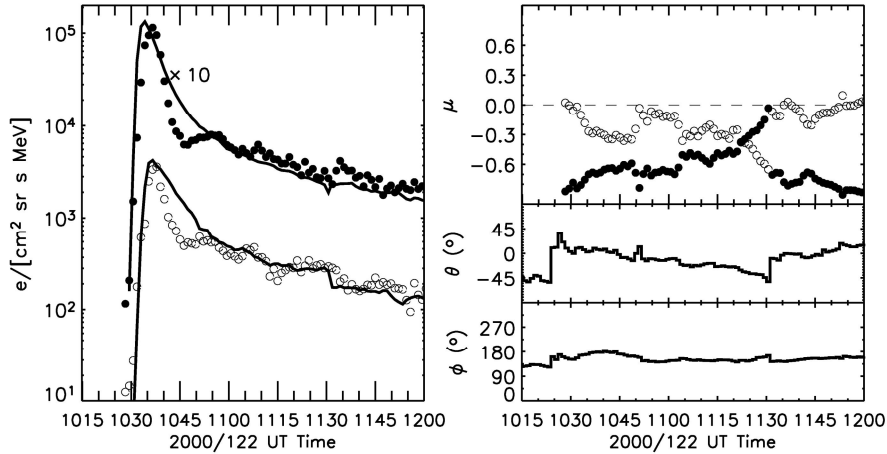


Fig. 1. Impulsive SEP event of 1 May 2000 as observed by ACE/EPAM. Electron intensities at 175–290 keV in two sectors of the instrument (left) and their zenith pitch-angle cosines (right: μ) with respect to the magnetic field direction (right: elevation θ , azimuth ϕ) as measured by ACE/MAG (Smith et al. 1998). On the left, the curves give the modeled intensities and the circles give the data.

The model fits the data satisfactorily at 1 AU. We only show one energy channel (175–312 keV) of four and two sectors of eight, but the other sectors and energy channels are taken into account in our fitting procedure as well and the quality of the fit is similar in all of them. The best-fit time scales of the rise and decay of the injection are 4.2 min and 1.2 min, respectively, and the radial **mean free path**, assumed to be independent of energy, is 0.6 AU.

We have investigated the time-intensity profiles of the modeled event at radial distances of 0.2 AU, 0.3 AU, 0.7 AU and 1 AU. We determined the time-integrated net-flux, the peak intensity, and the differential fluence of the event from the simulations (Fig. 2). The scaling of peak intensities is less steep than predicted by diffusion, which can be understood, because scatter-free transport corresponds to the scaling law $\propto \sec \psi / r^2$ and the event has a mean free path too long to be well described by diffusion. The scaling law of fluence, on the other hand, is steeper than the prediction of the diffusion-law (r^{-1} for a spatially constant radial mean free path), which

can be understood as well, because the result for scatter-free transport would again be $\propto \sec \psi / r^2$ for a solar source. The time-integrated net flux behaves like the diffusion theory predicts, as expected, because the same scaling law can be obtained from the focused transport equation (5).

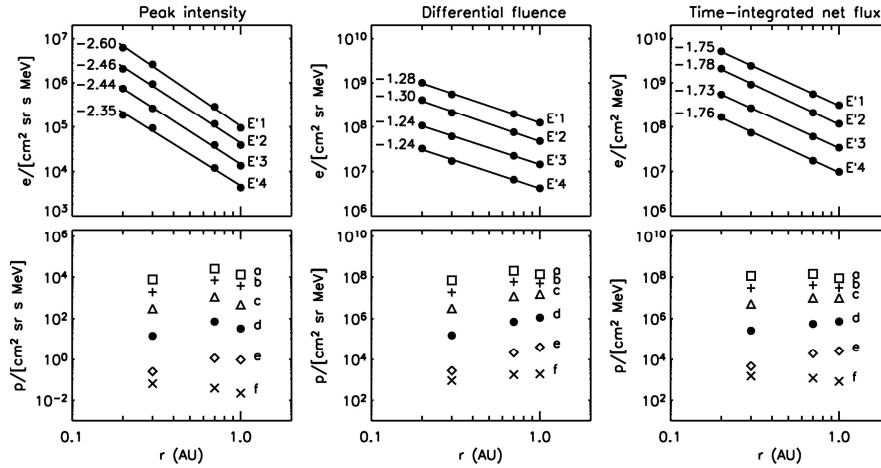


Fig. 2. Peak intensities, differential fluxes and time-integrated net flux of an impulsive electron event (upper panels) and a gradual proton event (lower panels) as a function of radial distance in several energy channels. The electron energy channels are E1': 45–62 keV, E2': 62–102 keV, E3': 102–175 keV, and E4': 175–312 keV. See Fig. 3 for a description of proton energy channels a-f.

Gradual SEP events

Gradual events in the IP space are more difficult to model than the impulsive ones for several reasons: (1) The source of the particles is the moving **shock front** driven by the CME through the IP medium; (2) the large spatial extent of the CME system and the long duration of the event means that transport conditions can vary during the event; and (3) the large intensities of the SEPs lead to a non-linear coupling between the accelerated particles and the plasma waves responsible for their scattering (e.g., Ng et al. 2003). Despite of these complications, dozens of gradual SEP events have been successfully modeled over the past two decades using an assumption of a particle source at the position of the IP shock and tracing the transport of the particles in the surrounding IP medium (Heras et al. 1992; Torsti et al. 1996; Lario et al. 1998; Aran et al. 2004, 2005).

As an example of gradual SEP event modeling, we consider the SEP event on 6–8 June 2000, observed by the ACE/EPAM at the L1 point, and by the CPME/IMP-8 (Sarris et al. 1976) orbiting the Earth (Fig. 3). The event was associated with a CME-driven shock that arrived at L1 at 08:41 UT of 8 June (DOY = 160.362). The CME was first observed by SOHO/LASCO C2 coronagraph on 6 June at 15:54 UT with an estimated speed of 1119 km s^{-1} . The associated X3.2/3B N20° E18° flare started at 14:58 UT June 6 (DOY = 158.624), marked by the arrow in Fig. 3.

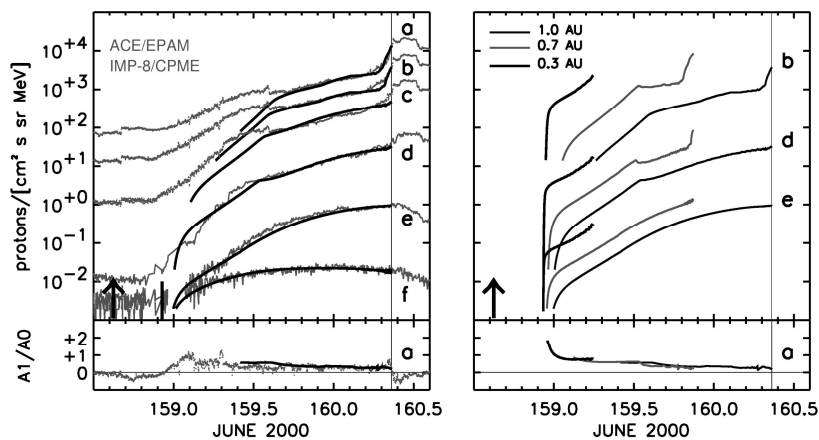


Fig. 3. Gradual SEP event of June 2000 as observed by ACE/EPAM and IMP-8/CPME. The observed and fitted intensities and anisotropies at 1 AU (left panel) are given in addition to the modeled ones at 0.3 AU and 0.7 AU (right panel). The energy channels are denoted with labels a–f: a: 0.58–1.06 MeV, b: 1.06–1.90 MeV, c: 1.90–4.80 MeV (from ACE/EPAM); d: 4.6–15.0 MeV, e: 15.0–25.0 MeV, f: 25.0–48.0 MeV (from IMP-8/CPME).

For eastern SEP events, modeling of the CME-driven shock evolution is essential to get an idea, when the observer obtains magnetic connection to the shock and, therefore, is able to observe the particles accelerated by the shock wave. We have simulated the propagation of the shock with the 2½D MHD code by Wu et al. (1983). Using the same functional form of the input pulse assumed by Smith and Dryer (1990), we inject a pulse centered at E18° with speed $V_s = 1138 \text{ km s}^{-1}$, angular width $\omega = 140^\circ$ and duration $\tau = 1 \text{ h}$. According to the simulation, ACE obtains the connection to the shock near the end of June 6, marked by the vertical line in Fig. 3.

SEP transport ahead of the shock is modeled by solving the focused transport equation using a finite difference method (Lario et al. 1998; Aran et al. 2005). The injection of particles at the shock is described by a semi-

empirical relation between the shock strength and injection rate. The mean free path in the upstream medium is taken to be $\lambda = \lambda_0(p/p_0)^{1/2}$, with the best fit values of $\lambda_0 = 0.1$ AU at $p_0 = (2m_p E_0)^{1/2}$ and $E_0 = 0.789$ MeV. A region just upstream the shock with a mean free path $\lambda = \lambda_0(p/p_0)^{-4/5}$ is assumed where $\lambda_0 = 0.01$ AU. This region starts to act at 21:00 UT and has a width that varies with the energy: 0.04 AU for $2 \text{ MeV} < E < 15 \text{ MeV}$, 0.06 AU for $1 \text{ MeV} < E < 2 \text{ MeV}$ and 0.07 AU for $E < 1 \text{ MeV}$. This region reproduces the effects that turbulence generated by the accelerated particles has on the particle transport close to the shock and allows us to fit the low-energy observations at the time of shock arrival. The fitted intensities and anisotropies are given in Fig. 3, along with the modeled intensities within the same flux tube at 0.3 and 0.7 AU.

The peak intensities, fluences, and time-integrated net fluxes of the event are given in Fig. 2. Note that the time integration of this event only covers the period before the shock passage. Clearly, the scaling obtained is very different from the impulsive event.

Modeling of IP particle acceleration

Particle acceleration in the IP medium occurs in shock waves formed in the compression regions between streams of different velocities. These include the forward and reverse shocks bounding the corotating interaction regions (CIRs) and the bow shocks of the fast CMEs. The CIR shocks are usually formed at radial distances 2–5 AU and are not capable of producing very energetic particle events in the inner heliosphere. Thus, the focus, from the point of view of space weather effects, is on the CME-driven shocks.

Physical modeling of particle acceleration is usually based on solving Parker's (1965) diffusion–convection transport equation with a source of low-energy particles placed at the shock. This model can be combined with the generation of MHD waves by the streaming accelerated particles self-consistently and solved in steady state and planar geometry (Bell 1978). Such an approach can be used to describe the local acceleration of low-energy (below 1 MeV) ions during times of IP shock passage (Lee 1983; Gordon et al. 1999), i.e., the so-called **energetic storm particle (ESP) events**. The same quasi-stationary particle acceleration model can be combined with a non-diffusive transport model to describe particle intensities at large distances upstream from the shock. This kind of approach has been used in both analytical (Lee 2005) and numerical (e.g., Rice et al. 2003) calculation of SEP event intensities in gradual events. While these physical

models give good insights to the particle acceleration processes in the IP medium, we do not yet know enough details of the acceleration mechanism and of the shock itself to allow detailed comparisons of theory and observations in individual gradual SEP events. Thus, transport modeling using phenomenological SEP source functions still remains an important tool for space weather studies.

Summary and outlook

We have reviewed the transport models used to describe the evolution of intensities and anisotropies during SEP events. In addition to giving fresh examples of typical modeling of impulsive and gradual SEP events, we used the models to calculate the peak intensities, the fluences and the time-integrated net fluxes of the events as a function of the radial distance from the Sun in the inner heliosphere. These quantities are the most important ones concerning the development of the solar corpuscular radiation environment as a function of distance from the Sun. The results were compared to the expectation derived from a simple analytical diffusion model.

The comparison shows that impulsive events show qualitatively similar scaling to the diffusion model, although at high values of the scattering mean free path, the scaling laws move closer to the scaling of scatter-free transport, as expected. The scaling of the gradual events, on the other hand, showed no similarities to the simple modeling. In our simulation, all the quantities under investigation showed more or less constant values as a function of radius, as a result of the interplay between geometry and time dependence of the source. On the other hand, we did not accurately model the non-linear coupling of the particles to the magnetic fluctuations responsible for their scattering in the IP medium (Ng et al. 2003). This effect may lead to completely different scaling laws: theoretical estimates predict that particle trapping close to the source is more efficient when the shock is close to the Sun (Vainio 2003). Thus, at small heliocentric distances the SEP events may be larger than close to 1 AU.

Our study demonstrates a pressing need for conducting more extensive modeling studies as well as analysis of observations at different distances from the Sun, to obtain reliable extensions of the present engineering models for SEP events like the model SOLPENCO (Aran et al. 2004).

Acknowledgements. We wish to thank B. Sanahuja for valuable discussions. We thank the ACE EPAM/SWEPAM/MAG teams for providing the ACE data used in this paper. We acknowledge the use of 330-s averaged IMP-8 CPME data available at the JHU/APL web site. RV acknowl-

edges financial support of COST-724. DL was partially supported by NASA grant NAG5-13487. NA and AA acknowledge the financial support of the Ministerio de Ciencia y Tecnología (Spain) under the Project AYA2004-03022 and partial computational support by the Centre de Supercomputació de Catalunya (CESCA).

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