Radial dependence of proton peak intensities and fluences in SEP events: Influence of the energetic particle transport parameters

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Abstract

We study the radial dependence of peak intensities and fluences of solar energetic particle (SEP) events in the framework of the focused transport theory. We solve the focused-diffusion transport equation that includes the effects of solar wind convection, adiabatic deceleration and pitch-angle scattering. We assume a Reid–Axford time profile for the particle injection at the base of a flux tube described by an Archimedean spiral magnetic field whose cross section \( A(r) \) expands as \( r^2 \cos(\psi(r)) \), where \( r \) is the radial distance and \( \psi(r) \) is the angle between the magnetic field line and the radial direction. We assume that energetic particles propagate along the field line. We locate several observers along the flux tube at radial distances ranging from 0.3 to 1.6 AU. Both peak intensities and event fluences decrease with increasing radial distance. We deduce functional forms to extrapolate peak fluxes and fluences with radial distance that depend on the energy of the particles, the pitch-angle scattering conditions, and the duration of the particle injection. The smaller the mean free path of the particles, the larger the decrease of both peak intensities and fluences with radial distance. The smaller the energy of the particles, the larger the decrease of both peak intensities and fluences with radial distance. Extended particle injections contribute to soften the decrease of the peak intensities with the radial distance but have no influence on the event fluence. We note that mobile particle sources (i.e. traveling interplanetary shocks) may vary the radial dependences deduced in this work.

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1. Introduction

In order to estimate the impact of solar energetic particles (SEPs) on interplanetary missions traveling at different radial distances it is necessary to specify the radial dependence of particle intensities and fluences. Observationally this is not an easy task because both radial and longitudinal effects are interrelated (Lario et al., 1998). Lario et al. (2006) used energetic particle data from the IMP-8 (at ~1 AU) and the two Helios spacecraft (from 0.3 to 0.98 AU) to analyze the peak intensities and fluences of SEP events observed simultaneously by at least two of these spacecraft at different radial distances. Lario et al. (2006) approximated the radial distributions of peak intensities and fluences of the ensemble of events by a functional form \( j = j_0 r^{-\alpha} \exp[-k(\Phi - \Phi_0)^2] \), where \( r \) is the heliocentric radial distance of the spacecraft, \( \Phi \) is the longitudinal angular distance between the footpoint of the field line connecting the observer to the Sun and the site of the active region that generated the event, \( \Phi_0 \) is the centroid of the distributions, and \( j \) is either the peak intensity or the event fluence. Lario et al. (2006) found that the dominant parameter that determines the peak intensity and fluence of the SEP events is not the heliocentric radial distance but rather the longitudinal distance between the parent active region and the footprint of the magnetic field line connecting the observer to the Sun. Lario et al. (2006) found that, over the ensemble of events, \( \alpha \) ranges from 2.7 to 1.9 for 4–13 and 27–
37 MeV proton peak intensities, and from 2.1 to 1.0 for 4–13 and 27–37 MeV proton event fluences.

Hamilton (1988) used the spherically symmetric transport model of Parker (1965) to study the radial dependence of peak intensities and fluences in SEP events. This transport model includes the effects of spatial diffusion, convection and adiabatic energy loss under the assumption that particle distributions are isotropic and that the focusing effect of energetic particles along the magnetic field is negligible. Both assumptions are not well supported by SEP observations in the innermost part of the heliosphere where the focusing effect is the dominant factor in the SEP transport. Large and long-lasting anisotropies are usually the focusing effect is the dominant factor in the SEP transport and adiabatic energy loss under the assumption that particle distributions are isotropic and that the focusing effect of energetic particles along the magnetic field is negligible. Both assumptions are not well supported by SEP observations in the innermost part of the heliosphere where the focusing effect is the dominant factor in the SEP transport.

In order to provide guidelines for flux extrapolation with radial distances, Hamilton (1988) used his transport model with the assumptions that (i) energetic particles propagate in a flux tube whose cross section expands as \( r^2 \), and (ii) the spatial radial diffusion coefficient is given by \( K_r = K_0 r^b \), where \( K_0 \) and \( b \) are parameters of the model (\( b < 2 \)). When the solar wind speed is set to \( V_{sw} = 0 \), this model reduces to a pure diffusion transport. In this case the transport equation may be solved analytically to find that the peak intensity follows a \( r^{-3} \) dependence (independent of \( K_0 \) or \( b \)) and the fluence follows a \( r^{-(b + 1)} \) dependence (if the cross section of the flux tube expands as \( r^2 \)). If \( V_{sw} \neq 0 \), the variation of the peak intensity with radial distance is more accentuated because of the effects of energy loss. Then, the variation with radial distance of peak intensities and fluences depends on both the values of \( b \) and \( K_0 \), as well as the energy spectrum of the particle intensities (Hamilton et al., 1990).

Application of the Hamilton (1988) model to proton measurements in the energy range of 10–70 MeV from 1 to 5 AU was used to obtain laws that allow extrapolation of particle intensities and fluences measured at 1 AU to other radial distances (Shea et al., 1988). In a workshop held at the Jet Propulsion Laboratory (JPL) in March 1987 on the Interplanetary Particle Environment, the working group consensus recommendations for radial extrapolation of peak fluxes and fluences, as documented in a JPL report edited by Feynman and Gabriel (1988), read as follows (see also Smart and Shea, 2003):

- Flux extrapolations from 1 to >1 AU: use a functional form of \( r^{-3.3} \) and expect variations ranging from \( r^{-4} \) to \( r^{-3} \).
- Flux extrapolations from 1 to <1 AU: use a functional form of \( r^{-3} \) and expect variations ranging from \( r^{-3} \) to \( r^{-2} \).
- Flux extrapolations from 1 AU to other distances; use a functional form of \( r^{-2.5} \) and expect variations ranging from \( r^{-3} \) to \( r^{-2} \).

These recommended power–law dependences are steeper than those deduced observationally by Lario et al. (2006) using data at \( r \leq 1 \) AU. The design of interplanetary missions planned to travel to the innermost part of the heliosphere such as the Inner Heliospheric Sentinels (Szabo, 2005) or Solar Orbiter (Marsden and Fleck, 2003) will benefit from the studies that estimate the radiation environment at different radial distances.

2. Radial dependences deduced from the focused-diffusion transport model

We solve the energetic particle transport equation deduced by Ruffolo (1995) that includes the effects of pitch-angle focusing in a diverging magnetic field, pitch-angle scattering due to magnetic field irregularities co-moving with the solar wind, solar wind convection and adiabatic deceleration. Details of the finite difference method used to solve numerically the transport equation can be found in Lario (1997). To study the influence of particle transport conditions on the radial gradients we have restricted ourselves to the case of a fixed particle source at the base of a flux tube. The inclusion of mobile particle sources (i.e. interplanetary traveling shocks) introduces a number of variables (e.g. shock speed, shock width, shock efficiency in particle acceleration, observer’s longitude with respect to the nose of the shock) that allows for a large variety of radial dependences (e.g. Aran et al., 2005; Vainio et al., 2006). Therefore, particle injection is assumed to take place at the base of an Archimedean spiral magnetic field. Solar wind speed is assumed to be \( V_{sw} = 431 \) km s\(^{-1}\), and an absorbent limit is set at \( r = 10 \) AU. Particle injection is assumed to follow a Reid–Axford profile as \( I(t) = N/\exp(-\beta t/t_s) \), where \( \beta \) and \( t_s \) are parameters that we will vary (Reid, 1964). The energy spectrum at the source is assumed to be \( p^{-4.8} \), where \( p \) is the particle momentum. We consider only protons with energies ranging from 4 to 48 MeV. We describe the pitch-angle scattering by a diffusion in the pitch-angle cosine \( \mu \) using a diffusion coefficient given by \( \phi(\mu) = C|\mu|^{a-1} (1 - \mu^2) \), where \( C = 3\pi/|2\lambda(4 - q)(2 - q)| \), \( v \) is the particle speed, \( \lambda \) is the mean free path along the magnetic field direction, and \( q \) is the spectral index of the power spectrum of magnetic field fluctuations that is set to a constant value \( q = 1.5 \). We set several observers at radial distances \( r = 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0, 1.2, 1.4, \) and 1.6 AU along the flux tube where energetic particles propagate.

We consider four different cases: Case 1 with \( \beta = 1.5 \) h, \( \tau = 0.2 \) h, and \( \lambda = 0.1 \) AU; Case 2 with \( \beta = 1.5 \) h, \( \tau = 1.5 \) h, and \( \lambda = 0.1 \) AU; Case 3 with \( \beta = 1.5 \) h, \( \tau = 0.2 \) h, and \( \lambda = 1.0 \) AU; Case 4 with \( \beta = 1.5 \) h, \( \tau = 1.5 \) h, and \( \lambda = 1.0 \) AU. In all these four cases, the mean free path \( \lambda \) is considered constant along the flux tube and the same value for all proton energies. Figs. 1–4a show for each one of these four cases the 8.3 MeV proton omnidirectional intensity as a function of time and as observed at dif-
different radial distances. The origin of time is set at the time when the particle injection starts. Solid circles indicate the maximum intensity of the plotted time-intensity profiles, whereas the open circles indicate the maximum intensity at the other distances not plotted in these figures. The adopted values for $b$, $s$, and $k$ are indicated in the figures.

The event fluence is computed as the integral over time of the omni-directional intensity after subtraction of a given value indicated by the horizontal dashed line in Figs. 1–4a. This value is set at three orders of magnitude below the peak intensity of the observer at $r = 1.6$ AU. The integration over time is extended until either 100 h or the time when intensities intersect the horizontal dashed line, whichever comes first. This base level is included just to represent a given background so that SEP intensities at 1.0 AU increase less than four orders of magnitude above the background level. The value adopted for this base level makes the computed fluence virtually insensitive to this subtraction.

Figs. 1–4b show the radial dependence of peak intensities (black dots) and event fluences (gray squares) for each one of the four cases we have considered. Owing to the decreasing magnetic field magnitude with increasing distance from the Sun, the focusing effect is more important within 1 AU than beyond. Therefore, we have fitted two power–laws $r^{-a}$ to both the peak intensities and fluences, one from 0.3 to 1.0 AU and the other from 1.0 to 1.6 AU. Table 1 lists the values of $a$ for the four cases considered in this study and for two different proton energies (8.3 and 34.6 MeV). Figs. 1–4b show that both peak intensities and fluences decrease with increasing radial distance. However, the radial dependence of this decrease depends on the parameters adopted to describe the particle injection ($\beta$ and $\tau$) and the transport conditions ($\lambda$).

Comparison of the Cases 1 and 2 with the Cases 3 and 4, respectively, shows that, with the same time-injection profile, the smaller the mean free path, the faster the decrease of peak intensities and fluences with radial distance (i.e. the larger the value of $s$). Comparison of the same Case but at the two different energies listed in Table 1 shows that, with the same time-injection profile and the same $\lambda$, the smaller the energy of the particles, the faster the decrease of peak intensity and fluences with radial distance (i.e. the larger the value of $a$). Comparison of the Cases 1 and 3 with the Cases 2 and 4, respectively, shows that with the same mean free path, the shorter the duration of the particle injection (i.e. the smaller the value of $\lambda$), the faster the decrease of the peak intensity with radial distance (i.e. the larger the value of $a$). The change of the particle injection duration, however, does not change significantly the variation with radial distance of the event fluence. Note that the subtraction of the base level (horizontal dashed line in Figs. 1–4a) may introduce a marginal variation of the event fluence with the shape of the injection profile. However, the fluences computed without this subtraction (but with the limit of the 100 h event duration) lead to radial dependences that are within the one-sigma error bars of those shown in Table 1. Therefore, the radial variation of the computed fluence does not depend on the shape of the injection profile as long as the rest of transport parameters are kept constant and the subtraction of the base level does not interfere the computation of the fluence.

Under scatter-free conditions and neglecting the effects of solar wind convection and adiabatic deceleration, the...
The intensity of a delta injection at the base of the flux tube decreases with radial distance as the inverse of the cross section of the flux tube. The section of a flux tube in an Archimedean magnetic field configuration varies with radial distance as $A(r) = r^2 \cos(\psi)$, where $\psi(r)$ is the angle between the magnetic field line and the radial direction ($\tan(\psi) = \Omega r / V_{sw}$, where $\Omega$ is the solar rotation angular velocity). By using the adopted value of $V_{sw}$ and following the same procedure to fit peak intensities and fluences described above, the inverse of $A(r)$ follows a power-law $r^{-1.74 \pm 0.03}$ from 0.3 to 1.0 AU and $r^{-1.37 \pm 0.02}$ from 1.0 to 1.6 AU. When scattering processes and solar wind effects are considered, peak intensities decrease faster with radial distance, while extended particle injections (as parameterized by $\beta$ and $\tau$) contribute to smoothen the decrease of peak intensities with radial distance. The combination of these two effects, even when $\lambda = 1.0$ AU, leads to values of $\alpha$ that are smaller than those predicted by a delta injection propagating under scatter-free conditions.

The values of $\alpha$ derived for 1.0–1.6 AU are larger than those derived for 0.3–1.0 AU (Table 1). Peak intensities, and hence fluences, decrease faster at large distances than close to the Sun. Processes of pitch-angle scattering compete with the focusing effect. Owing to the decreasing mag-

Fig. 2. The same as Fig. 1 but for Case 2: $\beta = 1.5$ h, $\tau = 1.5$ h, and $\lambda = 0.1$ AU.

Fig. 3. The same as Fig. 1 but for Case 3: $\beta = 1.5$ h, $\tau = 0.2$ h, and $\lambda = 1.0$ AU.
ngetic field magnitude with increasing distance from the Sun (first as $r^{-2}$ and as we move farther out from the Sun as $r^{-1}$), focusing effects are more important closer to the Sun, accounting for the different values of $\alpha$ from 0.3 to 1.0 AU and from 1.0 to 1.6 AU. In addition, for small mean free paths and short injections (Case 1), power–law indices for peak intensities can take values above three even for distances between 0.3 and 1.0 AU. Adiabatic deceleration processes, that become important when the mean free path is small and the energy of the particles is low, contribute to a decrease of peak intensities that is faster than that expected from a pure diffusion process.

The power–law indices listed in Table 1 are obtained by a numerical model based on a finite difference method and using specific parameters to describe particle injection, transport conditions and flux tube structure. Considering other transport parameters or even another finite difference scheme (with more numerical diffusion) may lead to slightly different values of $\alpha$.

Smart and Shea (2003) showed that SEP events where particle intensities are dominated by the effects of traveling interplanetary shocks, have intensities that do not scale in radial distance following simple power–law extrapolations. Models assuming the continuous contribution of particle acceleration by traveling shocks (Aran et al., 2005; Ruzmaikin et al., 2005; Vainio et al., 2006) obtain radial profiles of peak intensities and fluences that depend on both the energy of the particles and the parameters used in the model to characterize the shock formation, particle acceleration at the shock, and the transport of both SEPs and shocks. The heli-longitude of the parent solar event with respect to the observer’s location is the fundamental parameter that determines the radial variation of peak intensities and fluences (see details in Aran et al., 2005).

Table 1
Values of the power–law index $\alpha$ for each case

<table>
<thead>
<tr>
<th></th>
<th>8.3 MeV</th>
<th>34.6 MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.3–1.0 AU</td>
<td>1.0–1.6 AU</td>
</tr>
<tr>
<td>Case 1</td>
<td>$\beta = 1.5 , h$, $\tau = 0.2 , h$, $\lambda = 0.1 , AU$</td>
<td>$\beta = 1.5 , h$, $\tau = 0.2 , h$, $\lambda = 0.1 , AU$</td>
</tr>
<tr>
<td>Peak intensity</td>
<td>3.18 ± 0.05</td>
<td>3.60 ± 0.04</td>
</tr>
<tr>
<td>Event fluence</td>
<td>1.45 ± 0.03</td>
<td>2.00 ± 0.06</td>
</tr>
<tr>
<td>Case 2</td>
<td>$\beta = 1.5 , h$, $\tau = 1.5 , h$, $\lambda = 0.1 , AU$</td>
<td>$\beta = 1.5 , h$, $\tau = 1.5 , h$, $\lambda = 0.1 , AU$</td>
</tr>
<tr>
<td>Peak intensity</td>
<td>2.58 ± 0.08</td>
<td>3.49 ± 0.06</td>
</tr>
<tr>
<td>Event fluence</td>
<td>1.45 ± 0.03</td>
<td>2.01 ± 0.06</td>
</tr>
<tr>
<td>Case 3</td>
<td>$\beta = 1.5 , h$, $\tau = 0.2 , h$, $\lambda = 1.0 , AU$</td>
<td>$\beta = 1.5 , h$, $\tau = 0.2 , h$, $\lambda = 1.0 , AU$</td>
</tr>
<tr>
<td>Peak intensity</td>
<td>1.96 ± 0.06</td>
<td>3.06 ± 0.10</td>
</tr>
<tr>
<td>Event fluence</td>
<td>0.87 ± 0.02</td>
<td>1.00 ± 0.03</td>
</tr>
<tr>
<td>Case 4</td>
<td>$\beta = 1.5 , h$, $\tau = 1.5 , h$, $\lambda = 1.0 , AU$</td>
<td>$\beta = 1.5 , h$, $\tau = 1.5 , h$, $\lambda = 1.0 , AU$</td>
</tr>
<tr>
<td>Peak intensity</td>
<td>1.56 ± 0.03</td>
<td>2.34 ± 0.09</td>
</tr>
<tr>
<td>Event fluence</td>
<td>0.86 ± 0.02</td>
<td>0.99 ± 0.03</td>
</tr>
</tbody>
</table>

Fig. 4. The same as Fig. 1 but for Case 4: $\beta = 1.5 \, h$, $\tau = 1.5 \, h$, and $\lambda = 1.0 \, AU$. Please cite this article in press as: Lario, D. et al., Radial dependence of proton peak intensities and fluences ..., J. Adv. Space Res. (2007), doi:10.1016/j.asr.2007.01.057
3. Conclusions

Transport processes undergone by energetic particles as they propagate away from the Sun contribute to the decrease of peak intensities and fluences with radial distance. Pitch-angle scattering and adiabatic deceleration processes are dominant over the focusing and solar wind convection effects at low energies, large heliocentric distances and when mean free paths are small. Hence, the large values of $x$ deduced for the distances 1.0–1.6 AU at low energies and with small mean free paths. The dependence of the power–law indices $x$ with the energy of the particles and the adopted values of $\lambda$ shows that (i) the smaller the mean free path of the particles, the larger the decrease of both peak intensities and fluences with radial distance, and (ii) the smaller the energy of the particles, the larger the decrease of both peak intensities and fluences with radial distance. The radial variation of the focusing effect leads to larger values of $x$ for distances between 1.0 and 1.6 AU than for distances between 0.3 and 1.0 AU. When particle injection at the base of the flux tube extends over a long time interval, peak intensities do not decrease so fast with radial distance as when particle injections are of short duration. The radial dependence of the total event fluence does not vary with the duration of the particle injection as long as the rest of transport parameters are kept constant.

The power–law dependences derived from the focused-diffusion transport equation with a fixed source close to the Sun are, in general, gentler than those recommended for radial extrapolation of intensities observed at 1 AU (Feynman and Gabriel, 1988), especially within 1 AU of the Sun. The values of $x$ listed in Table 1 are closer to those deduced observationally by Lario et al. (2006) using data from 0.3 to 1.0 AU, suggesting that the framework of the focused-diffusion transport is more appropriate to describe particle propagation than a pure-diffusion model, especially within 1 AU of the Sun. The steeper recommended radial extrapolations (Feynman and Gabriel, 1988) can be used to design instrumentation for missions traveling within 1 AU allowing engineers to be on the safe side. Then, guidelines to extrapolate particle intensities from 1 AU to other heliocentric radial distances should be given as worst-case limits rather than absolute laws valid for all cases and scenarios. The power–law dependences deduced observationally (Lario et al., 2006) and those deduced here may relax the limiting restrictions used to design particle detectors for inner heliospheric missions.

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