Particle acceleration at perpendicular shock waves: Model and observations

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Received 7 November 2005; revised 17 February 2006; accepted 27 February 2006; published 23 June 2006.

On the basis of a recently developed nonlinear guiding center theory for the perpendicular spatial diffusion coefficient $k_\perp$ used to describe the transport of energetic particles, we construct a model for diffusive particle acceleration at highly perpendicular shocks, i.e., shocks whose upstream magnetic field is almost orthogonal to the shock normal. We use $k_\perp$ to investigate energetic particle anisotropy and injection energy at shocks of all obliquities, finding that at 1 AU, for example, parallel and perpendicular shocks can inject protons with equal facility. It is only at highly perpendicular shocks that very high injection energies are necessary. Similar results hold for the termination shock. Furthermore, the inclusion of self-consistent wave excitation at quasiparallel shocks in evaluating the particle acceleration timescale ensures that it is significantly smaller than that for highly perpendicular shocks at low to intermediate energies and comparable at high energies. Thus higher proton energies are achieved at quasiparallel rather than highly perpendicular interplanetary shocks within 1 AU. However, both injection energy and the acceleration timescale at highly perpendicular shocks are sensitive to assumptions about the ratio of the two-dimensional (2-D) correlation length scale to the slab correlation length scale $\lambda_{2D}/\lambda_k$. Model proton spectra and intensity profiles accelerated by a highly perpendicular interplanetary shock are compared to an identical but parallel interplanetary shock, revealing important distinctions. Finally, we present observations of highly perpendicular interplanetary shocks that show that the absence of upstream wave activity does not inhibit particle acceleration at a perpendicular shock. The accelerated particle distributions closely resemble those expected of diffusive shock acceleration, and observed at oblique shocks, an example of which is shown.


1. Introduction

[2] Particle acceleration is ubiquitous at shock waves, occurring on scales ranging from supernova remnants to interplanetary shocks and cometary bow shocks. The mechanism thought to be responsible for the almost universally observed power-law spectra is diffusive shock acceleration [Axford et al., 1977; Bell, 1978a, 1978b; Blandford and Ostriker, 1978; Krymsky, 1977]. The observed cosmic ray spectrum, until about $10^{14}$ eV/nuc., can be explained by particle acceleration at a supernova remnant shock [e.g., Volk et al., 1988]. Diffusive shock acceleration is also generally thought to account for gradual SEP events [e.g., Reames, 1999], but the correspondence between the simple predictions of theory and observations is often not compel-ling [e.g., Desai et al., 2003] and more elaborate time-dependent models have had to be developed [Zank et al., 2000; Li et al., 2003, 2005b; Rice et al., 2003; Li and Zank, 2005].

[3] The importance of particle acceleration at perpendicular shocks has been recognized in the context of understanding both solar energetic particle (SEP) events (“shocks near the Sun”) and energetic storm particle (ESP) events (“shocks at 1 AU”). Tylka et al. [2005] suggest that the following characteristics seem to group events. For Group I, the Fe/O ratio falls with energy and is Fe-poor at high energies, and is a bigger event of longer duration, with a softer spectrum and exponential rollover. For Group II, the Fe/O ratio increases with energy and is Fe-rich at high energies, and is a smaller event of shorter duration with a harder spectrum that is nearly power-law and no rollover. To explain this grouping of events, Tylka et al. [2005] invoke particle acceleration either at quasiparallel or quasiperpendicular shocks. However, Desai et al. [2004] find that the distinction between these groups is less sharp when many events are considered.

[4] While the theory of particle acceleration at a quasiparallel shock appears to be reasonably well understood,
and has been applied to SEP and ESP events [e.g., Zank et al., 2000; Li et al., 2003, 2005b], no similar theory exists for perpendicular shocks. Here we describe an approach for diffusive shock acceleration at perpendicular shocks. Besides interplanetary shocks, this topic is of considerable importance to the heliospheric termination shock since it is likely to be highly perpendicular across most of its surface.

A related issue is of course the question of injection, which has attracted considerable attention in the past.

[5] Near an oblique shock front, Alfvén waves are responsible for particle scattering. Locally, at the simplest level, the particle distribution function and forward and backward wave energy densities \( I_{\pm} \), normalized to \( B^2/(8\pi) \), per logarithmic bandwidth are coupled together through the 1D transport equations for the particles and waves [Lee, 1983; Bell, 1978a, 1978b; Gordon et al., 1999],

\[
\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} - \frac{p}{3} \frac{\partial f}{\partial x} \left[ \kappa_{\parallel}(p) \frac{\partial f}{\partial x} \right] = 0,
\]

[1]

\[
\frac{\partial I_{\pm}}{\partial t}(k, t) + (u \pm V_A) \frac{\partial I_{\pm}}{\partial x}(k, t) = \Gamma_{\pm} I_{\pm}(k, t) - \gamma_{\pm} I_{\pm}(k, t),
\]

[2]

\[
\kappa_{\parallel}(p) = \frac{\kappa_0}{(I_{\pm} + I_{||})(k) B} \frac{(p/p_0)^2}{\sqrt{(m_p c^2/p_0)^2 + (p/p_0)^2}};
\]

[3]

where \( \Gamma_{\pm} \) and \( \gamma_{\pm} \) are the growth and damping rates of \( I_{\pm} \), \( \kappa_{\parallel} \) the spatial diffusion coefficient, \( \kappa_0 \) the Bohm limit, \( u \) the flow speed, \( B \) the (interplanetary) magnetic field, and \( p \) the particle momentum. Also, the wave number \( k = r_L^{-1} \), the particle gyroradius, and \( p_0 \) is a constant momentum scale and \( B_0 \) is a normalizing magnetic field (taken to be the interplanetary magnetic field at the co-rotation radius [see Zank et al., 2000]). As we discuss below, equations (1)–(3) are appropriate to oblique shocks that are not very close to perpendicular. In this case, the particle transport equation and the wave energy density equation are coupled through the spatial diffusion tensor and the wave driving term \( \Gamma_{\pm} \), and these equations must be solved simultaneously to determine both the accelerated particle spectrum (responsible for generating the waves) and the wave energy density (responsible for scattering the particles) self-consistently.

(However, the wave intensity driven by the accelerated particle gradient at a strong shock can often exceed the background mean magnetic field intensity, which violates the quasilinear assumption implicit in equation (1). In this case, the Bohm limit is typically applied; that is, \( \kappa_0 \) is used when the wave energy density per logarithmic bandwidth exceeds the local solar wind magnetic energy density [Zank et al., 2000; Rice et al. [2003] and Li et al. [2003] use the work of Gordon et al. [1999] to evaluate the wave intensity from (2). Steady solutions to (1) show that the accelerated particle intensities are constant downstream of the oblique shock and exponentially decaying upstream of the shock. The scale length of the decay is determined by the momentum dependent diffusion coefficient (steady state solution). Downstream and away from the shock, we need to include the radial expansion of the solar wind. The approach we use follows that of Zank et al. [2000] and Li et al. [2003]. Thus particles trapped by the turbulence convected from upstream of the shock experience convection, cooling as the flow expands back to its ambient state, and of course diffusion. Owing to imperfect trapping upstream and downstream of the shock, particles can escape upstream from the shock into the undisturbed solar wind. The above is illustrated schematically in Figure 1. The escaping particles stream along the IMF, experiencing some scattering, and are observed at 1 AU as gradual SEP events. Observationally, waves are often detected at quasiparallel shocks [e.g., Tsurutani et al., 1983; Bamert et al., 2004].

[6] However, particle acceleration via the first-order Fermi mechanism at a perpendicular shock wave remains an outstanding problem for two essential reasons. The first is that quasiparallel shocks, accelerated particles at a perpendicular shock cannot excite the (Alfvén) wave field that is responsible for scattering the particles repeatedly across the shock. This is because the growth term in the wave equation, besides being proportional to the particle gradient, is proportional to \( \cos \theta_{bn} \), where \( \theta_{bn} \) is the angle between the upstream magnetic field and the shock normal direction, and is thus zero for a strictly perpendicular shock [e.g., Gordon et al., 1999; Li et al., 2003; Rice et al., 2003]. Unlike the particle acceleration model described by (1)–(3), this therefore requires that particle scattering at a perpendicular shock be the result of in situ upstream turbulence that is convected into the shock. At the heart of the problem for perpendicular shocks is the need for a viable model of the perpendicular component of the diffusion tensor. This is discussed in greater detail in section 2. The second problem is that since an accelerated particle is essentially tied to a magnetic field line, its ability to cross the shock repeatedly is limited to the time it takes a magnetic field line to cross from upstream to downstream, assuming that the magnetic field line experiences some wandering (see below) to make
the transmission time nonzero. Thus a fast-moving particle is necessary if it is to experience multiple crossings of the perpendicular shock (see Figure 2) so that it can be diffusively accelerated. Consequently, diffusive shock acceleration of particles at a perpendicular shock is effective for particles that are already energetic. This is referred to as the “injection problem” for perpendicular shocks [Jokipii, 1987; Zank et al., 1996b]. A more formal approach to the injection problem is discussed below. However, it is generally thought that the acceleration time at a perpendicular shock is much faster than at a parallel shock since the acceleration timescale,

$$\tau_{acc} = \frac{(1/dp)}{p} \frac{1}{V_{sh}(s-1)},$$

(4)

(where $p$ is particle momentum, $\kappa_{xx}$ the spatial diffusion coefficient normal to the shock, $s$ the compression ratio, and $V_{sh}$ the upstream shock flow speed) is proportional to the particle spatial diffusion coefficient. Suppose we assume a general form for the diffusion coefficient parallel to the magnetic field, $\kappa_{||} = v\lambda_{||}/3$. In the hard sphere scattering approximation, we may express the perpendicular diffusion coefficient as $\kappa_{\perp} = \kappa_{||}(1 + \nu^2)$, [e.g., Jokipii, 1987] where $\nu$ is the gyroradius, $v$ the particle velocity, $\lambda_{||} = 3\kappa_{||}/v$, and $\nu = \lambda_{||}/r_L$ is a measure of the particle scattering strength. In the weak scattering limit, $\lambda_{||}/r_L \gg 1$ and $\kappa_{||}/\kappa_{\perp} = (\lambda_{||}/r_L)^{-2} \ll 1$. The strong scattering limit is defined as $\lambda_{||}/r_L \ll 1$, which corresponds to $\langle dB^2 \rangle \approx B^2$ (strong turbulence). This theory is not applicable in the case when $\lambda_{||}/r_L \ll 1$, because when a particle scatters many times during one gyro-orbit, its cyclotron radius no longer has meaning. In the case when $\lambda_{||}/r_L \simeq 1$, we get $\kappa_{\perp} \simeq \kappa_{||}$, which is in fact the Bohm limit (isotropic diffusion) [Forman et al., 1974; Jokipii, 1987]. Clearly, for weak scattering in the hard sphere approximation, the acceleration time at a quasiparallel shock ($\kappa_{xx} \simeq \kappa_{||}$) is much larger than at a quasiperpendicular shock ($\kappa_{\perp} \ll \kappa_{||}$). This result is however predicated on the assumption that the intensity of the upstream turbulence is the same at a parallel and perpendicular shock, which, as we have seen above, is incorrect because waves are excited at a quasiparallel shock and not at a quasiperpendicular shock. A physically meaningful comparison of acceleration timescales must take this into account.

2. Perpendicular Diffusion Coefficient

The problem of perpendicular diffusion by a particle in a turbulent plasma is a problem of enduring interest, and one that has yet to be fully solved. Theoretically, particle diffusion parallel to the magnetic field appears to be described well (at least for pure pitch-angle scattering in low-frequency MHD or slab turbulence) by a quasi-linear theory based on the resonant wave-particle interaction. By contrast, theoretical models for $\kappa_{\perp}$ do not appear to describe observations [e.g., Mazur et al., 2000; Fisk et al., 1997], nor are they consistent with careful numerical simulations [Giacalone and Jokipii, 1999; Mace et al., 2000; Qin et al., 2002a, 2002b]. Quasi-linear theory suggests that perpendicular diffusion of particles is primarily the result of charged particles following magnetic field lines that are themselves experiencing a random walk in space and time, the Field Line Random Walk (FLRW) model, but numerical simulations with low-energy particles do not support this simple, intuitive, and appealing approach. Matthaeus et al. [2003] introduced a theoretical model which, for the first time, appears to be consistent with numerical simulations in both the high- and low-energy particle regimes. Their approach is to assume that perpendicular transport is governed by the velocity of gyrocenters that follow magnetic field lines. Qin et al. [2002a, 2002b] showed using numerical simulations that perpendicular diffusion could occur only in the presence of a transverse complex magnetic field; that is, if field lines sampled by the gyromotion of a particle experiencing parallel and antiparallel scattering along the field are similar to those encountered before a scattering event, then perpendicular diffusion is suppressed. Although a more precise definition (and explicit examples) is given by Matthaeus et al. [1995, 2003], flux surfaces with high transverse complexity are characterized by the rapid and random separation of nearby magnetic field lines and are therefore important to perpendicular diffusion. In particular, it appears that the combination of slab and 2-D turbulence (a “two-component” model) is necessary to produce transverse complexity, and that slab turbulence alone, for example, is insufficient. [8] Zank et al. [2004] present a derivation showing (1) that a superposition of slab and 2-D turbulence is necessary for the development of transverse complexity in the magnetic field, and (2) that the transverse complexity can be treated as diffusive. By assuming that magnetic field lines separate diffusively, Matthaeus et al. [2003] derive a nonlinear theory for the perpendicular diffusion coefficient, which corresponds to a solution of the integral equation

$$\kappa_{\perp} = \frac{a^2 v^2}{3B^2} \int_0^\infty \frac{S_{A}(k)dk}{v/\lambda_{||} + k^2\kappa_{\perp} + k^2\kappa_{||}},$$

(5)
In (5), $B$ is the mean magnetic field strength, $\lambda_{||} = 3\kappa_{||}/v$ is the mean free path (mfp) parallel to the mean magnetic field (and $\lambda_{\perp} = 3\kappa_{\perp}/v$ the corresponding perpendicular mean free path), $k_{||,\perp}$ the wave vector parallel/perpendicular to the mean magnetic field, and $a^a$ is a factor related to the gyrocenter velocity and is found numerically to be $\sim 1/3$ [Matthaeus et al., 2003]. Expression (5) also makes the simplifying assumption of magnetostatic turbulence. The extension to include dynamical turbulence is straightforward. The spectral amplitude of the turbulent fluctuations is given by $S_{\perp}(k)$. The model described by equation (5) is referred to as the Nonlinear Guiding Center (NLGC) theory.

[Zank et al. 2004] introduce an explicit model for turbulence in the solar wind and solve (5) approximately for $\lambda_{\perp}$. Theory [Zank and Matthaeus, 1992] and observations [Matthaeus et al., 1990; Bieber et al., 1996] suggest that turbulence in the solar wind is comprised of a superimposed slab and 2-D component. The two-component slab-2-D model ignores the usually smaller parallel variance and includes only fluctuations with wave vectors either purely parallel ($k_\parallel$) to or perpendicular ($k_\perp$) to the mean magnetic field $B_0$. Thus we may express

$$S_{\perp}(k) = S_{\perp}^{2D}(k_\parallel) b(k_\parallel) + S_{\perp}^{slab}(k_\parallel) b(k_\parallel),$$

(6)

where we choose

$$S_{\perp}^{2D}(k_\parallel) = C(\nu) \left( \frac{B_{\parallel,0}}{B_0} \right)^{1/2} \left( \frac{1 + k_\parallel^2 \lambda_{2D}^2}{k_\parallel} \right)^{-\nu},$$

(7)

$$S_{\perp}^{slab}(k_\parallel) = C \left( \frac{B_{\parallel,0}}{B_0} \right)^{1/2} \left( \frac{1 + k_\parallel^2 \lambda_{slab}^2}{k_\parallel} \right)^{-\nu},$$

(8)

and $C(\nu) = \frac{1}{2\nu^2} \Gamma(\nu)/\Gamma(\nu - 1/2)$ is a constant related to the spectral index $2\nu$. We assume that the spectrum has an inertial range that is characteristic of fully developed Kolmogorov turbulence and thus $\nu = 5/6$. The remaining variables in (7) and (8) are the ensemble-averaged energy density in slab/2-D magnetic fluctuations, $\langle b_{\parallel,0}^2 \rangle$ and $\langle b_{\parallel,0}^2 \rangle$, and the spectral “bend-over” scales in the $||$- and $\perp$-directions for slab/2-D turbulence, are denoted by $\lambda_{slab}$ and $\lambda_{2D}$. The bend-over scales are related to the slab and 2-D correlation lengths $\lambda_{slab}^2$, $\lambda_{2D}^2$, respectively. In the solar wind, $\lambda_{slab} > \lambda_{2D}$ for fully evolved turbulence [Dasso et al., 2005].

An approximate expression for the perpendicular diffusion coefficient $\kappa_{\perp}$ can be derived from (5) [Zank et al., 2004], yielding

$$\lambda_{\perp} \approx \sqrt{3} \pi a^a C \left( \frac{B_{\parallel,0}}{B_0} \right)^{1/2} \left[ \frac{\langle b_{\parallel,0}^2 \rangle}{\langle b_{\parallel,0}^2 \rangle} \right]^{1/2} \min(\lambda_{slab}, \lambda_{\perp}/\sqrt{3}) \times \left[ 1 + \left( \frac{a^a C}{\sqrt{3} \pi} \right)^{1/3} \left( \frac{\langle b_{\parallel,0}^2 \rangle}{\langle b_{\parallel,0}^2 \rangle} \right)^{1/2} \lambda_{2D}^2 \right]^{1/3} \times \left( 4.332 \left( \lambda_{slab}^2 - \left( \frac{\lambda_{\perp}}{\sqrt{3}} \right) \right) + 3.091 H \left( \frac{\lambda_{\perp}}{\sqrt{3}} - \lambda_{slab} \right) \right)^{2/3}.$$  

(9)

The dependence of the diffusive mean free path $\lambda_{\perp}$ on particle energy or gyroradius $r_L$ enters only through the parallel diffusion coefficient or mean free path $\lambda_{||}$. In the quasilinear form, $\lambda_{||}$ is well approximated by

$$\frac{\lambda_{||}}{\lambda_{slab}} = 2.343 \left( \frac{B_{\parallel,0}^2}{\langle b_{\parallel,0}^2 \rangle} \right) \left( \frac{r_L}{\lambda_{slab}} \right)^{1/3} (1 + D),$$

$$D = \frac{7/4}{(m + 1/3)(m + 7/3)},$$

$$A = (1 + n^2)^{5/6} - 1,$$

$$n \equiv 0.746834 r_L/\lambda_{slab}^2,$$

$$m = \frac{5n^2/3}{1 + n^2 - (1 + n^2)^{1/6}}.$$

Although an approximation, expression (10) is very close to the exact Fokker-Planck result [Zank et al., 1998]. The energy dependence of $\lambda_{\perp}$ enters through the parallel mean free path (mfp). Since $\lambda_{\perp} \propto f_{\perp,L}^{1/3}$ to leading order, it therefore has only a very weak energy dependence. As in our assumption about (5), effects associated with dynamical MHD turbulence were neglected in deriving (10). The term $D$ can be important for very energetic particles whose Larmor radius is comparable to or greater than the correlation length $\lambda_{slab}$, since the particle then scatters resonantly with the flatter energy-containing range rather than the inertial range. This is of particular importance in the outer heliosphere [Zank et al., 1998].

[11] The approximate solution (9) is in excellent agreement with the exact solution of the integral equation. The physical content of the NLGC theory is revealed clearly by the approximate solution, showing how $\kappa_{\perp}$ scales with parameters such as the energy density in magnetic fluctuations $\langle b_{\parallel,0}^2 \rangle$, mean field strength, particle gyroradius, MHD turbulence correlation length scales, parallel diffusion coefficient, etc. This dependence is illustrated in Figure 3 (left plot), which shows the parallel and perpendicular mfps and the particle gyroradius as a function of energy for the examples of a perpendicular heliospheric termination shock located at 100 AU at low latitudes and an interplanetary shock located at 1 AU. To evaluate the expected level of turbulence at the termination shock, we have used the turbulence transport model of Zank et al. [1996a]. Another parameter that is needed to determine $\kappa_{\perp}$ is the ratio $\lambda_{2D}/\lambda_{slab}$, for which definite values are not known and it may be as little as 0.1 [Zank et al., 1998]. For Figure 3, we use ratio of 0.5. Notice that at both 100 AU and at 1 AU, the form of $\lambda_{\parallel}$ and $\lambda_{\perp}$ is very similar over the energy range 1 keV to 100 MeV (especially at 1 AU). For the termination shock example, $\lambda_{\perp}/\lambda_{\parallel} \approx 10^{-5}$ at low energies and $10^{-4}$ at 100 MeV. Figure 3 (right plot), plotted with a different normalization, shows the parallel and perpendicular mfps and Bohm diffusion coefficient at 1 AU instead. Figure 3 can also be regarded as a plot of the acceleration timescale for the NLGC diffusion coefficient, the quasilinear parallel diffusion coefficient, and the Bohm diffusion coefficient, all normalized to the Bohm acceleration timescale limit; that is, we plot $\tau_{acc}/\tau_{Bohm} = \kappa_{||}/D_{||}$, as a function of gyroradii normalized to the turbulence correlation length $r_L/\lambda_{\parallel}$. Obviously, the Bohm acceleration time is constant (unity) in such a plot. The parallel and perpendicular acceleration
timescales, when normalized to the Bohm acceleration timescale, decrease with increasing energy since $\lambda_{||} \propto r_{\perp}^{1/3}$ in the inertial range. For the typical solar wind parameters used in Figure 3 (right), the acceleration timescale for a perpendicular shock is more than 3 orders of magnitude smaller than that at a parallel shock. For the 1 AU shock, we used a magnetic field magnitude of 4.12 nT, which is what was used by Zank et al. [2004]. The ratios $\langle b^2 \rangle / B^2 = 0.78$ and $\lambda_{2D}/\lambda_{slab} = 0.5$ are used here and in Figure 5 in section 3. The magnitude of the slab correlation length ($\lambda_{slab}$) was assumed to be 0.03 AU, and the 2-D-slab ratio is 80:20. Finally, the solar wind speed is taken to be 400 km/s and the shock compression ratio to be 3.0.

[12] Several key points are revealed by the graphs. The first is that the parallel and perpendicular diffusion coefficients track each other rather closely and $\kappa_{\perp} \sim 0.01 - 0.001 \kappa_{||}$. A second is that the perpendicular diffusion coefficient is larger than the Bohm limit at low energies and significantly smaller at high energies. Thus the acceleration timescale at a perpendicular shock is much faster than at a parallel shock, assuming equal levels of upstream turbulence at each shock. The perpendicular mfp at 100 AU is not significantly different from that at 1 AU, especially at high energies, implying that acceleration timescales for perpendicular shocks are similar throughout the heliosphere. Consequently, perpendicular shock acceleration may be common in the outer heliosphere. By contrast, nearly 2 orders of magnitude separate the parallel mfp at 100 AU and 1 AU.

3. Anisotropy and Injection

[13] As discussed in section 1 and reflected in Figure 2, for particles to be accelerated diffusively at a quasiperpendicular shock requires that they be sufficiently energetic already. Our physical intuition, which is reflected in Figure 2, does not easily allow us to compute the energy a particle needs in order to be “injected” into the diffusive acceleration process. In order to apply the cosmic ray transport equation to diffusive shock acceleration, we require the particle anisotropy to be small at the shock. Assume that $yz$ is the plane of the shock, and that the magnetic field vector lies in the $xz$ plane and makes an angle $\theta_{bn}$ with the shock normal ($x$), which is coincident with the direction of the flow $u$. If we assume that the distribution function $f$ depends on $x$ only and consider the distribution in the shock frame, the streaming flux components are

$$F_x = -\frac{4\pi p^2}{v} \left( \kappa_{xx} \frac{\partial f}{\partial x} + u p \frac{\partial f}{\partial p} \right),$$

$$F_y = \frac{4\pi p^2}{v} \kappa_{yy} \frac{\partial f}{\partial y},$$

$$F_z = \frac{4\pi p^2}{v} \kappa_{zz} \frac{\partial f}{\partial z},$$

where $v$ is the particle speed. For a planar shock

$$\frac{\partial f}{\partial x} = \frac{uf}{\kappa_{xx}}, \quad p \frac{\partial f}{\partial p} = -q f,$$

where $q = 3s/(s - 1)$ is the power law index of the accelerated spectrum and $s$ is the shock compression ratio as before. The diffusion tensor components are

$$\kappa_{xx} = \kappa_{\perp} \sin^2 \theta_{bn} + \kappa_{||} \cos^2 \theta_{bn},$$

$$\kappa_{yy} = -\kappa_{d} \sin \theta_{bn},$$

$$\kappa_{zz} = (\kappa_{||} - \kappa_{\perp}) \sin \theta_{bn} \cos \theta_{bn}.$$
and the classical coefficient $\kappa_{Bohm} = r_v V/3$ is used below. The total anisotropy is $\xi = 3 |F|/(4\pi I)$, where $J = fp^2$ is the differential intensity. Since $\kappa_\parallel \ll \kappa_\perp$, we obtain

$$\xi = \frac{3u}{v} \left[ \left( \frac{q}{3} - 1 \right)^2 \left( \frac{\kappa_{Bohm} + \kappa_\perp^2 \cos^2 \theta_{inj}}{\kappa_\parallel \sin^2 \theta_{inj} + \kappa_\perp \cos^2 \theta_{inj}} \right)^{1/2} \right]. \quad (16)$$

For a nearly perpendicular shock, $\sin \theta_{inj} \approx 1$, so that

$$\xi = \frac{3u}{v} \left[ \frac{1}{(s - 1)^2} \left( \frac{r_v^2 + \lambda_2^2 \cos^2 \theta_{inj}}{(\lambda_{slab} + \lambda_1 \cos^2 \theta_{inj})^2} \right)^{1/2} \right]. \quad (17)$$

For the diffusion approximation to be valid, we require that the anisotropy $\xi$ be small, i.e., $\xi \ll 1$.

Some very interesting results emerge when one computes the anisotropy as a function of energy for shocks of different obliquities. Alternatively, by assuming that $\xi = 1$, we can compute the injection energy threshold as a function of shock obliquity. It should be noted that our assumption of $\xi = 1$ is for computational convenience only and that $\xi \ll 1$ is really required for the diffusion approximation to be valid. The use of $\xi = 1$ provides an upper limit from which we can scale results if necessary. Related discussions have been presented by Giacalone and Jokipii [1999, 2005]. We again consider two examples: the termination shock at 100 AU (Figure 4) and an interplanetary or CME-driven shock within 1 AU (Figure 5).

The figures reveal two interesting results.

1. The anisotropy is very sensitive (and singular) in the limit as $\theta_{inj} \rightarrow 90^\circ$. The sensitivity to the angle between the magnetic field $B$ and the shock normal in the limit of $\theta_{inj} \rightarrow 90^\circ$ is because of the very large contribution from the parallel diffusion coefficient $\lambda_\parallel$. For low energies, the anisotropy $\xi$ is approximately equal to $\theta_{inj} = 0^\circ$ and $\theta_{inj} = 90^\circ$ but an order of magnitude different for $\theta_{inj} = 89^\circ$. For $\theta_{inj} \leq 89^\circ$, $\xi$ decreases approximately as a power law in energy $T$, whereas the $\theta_{inj} = 90^\circ$ curve has a distinct plateau at intermediate energies before decreasing slowly. The corresponding anisotropy figure for parameters appropriate to 1 AU (Figure 5, left panel) shows that there is virtually no difference between $\theta_{inj} = 0^\circ$ and $\theta_{inj} = 90^\circ$. With increasing obliquity, the anisotropy differences become more marked. Figures 4 and 5 suggest that highly oblique shocks rather than perpendicular shocks will have the most difficulty injecting particles into the diffusive shock acceleration mechanism.

2. By plotting the injection threshold energy $T_{inj}$ (defined as the energy at which $\xi = 1$) as a function of $\theta_{inj}$ (Figures 4 and 5), we can see explicitly that highly oblique shocks require the highest injection energies. For the termination shock example, the injection energy $T_{inj}$ increases rapidly with increasing obliquity by nearly 3 orders of magnitude ($\sim 2$ keV at a parallel termination shock (Figure 4) to $>1$ MeV for $\theta_{inj} \simeq 88^\circ$). For the termination shock (Figure 4, right panel), the exact magnitude and the location of the maximum depend quite sensitively on the value of the ratio $\lambda_{2D}/\lambda_{slab}$. The frequently quoted value of $\lambda_{2D}/\lambda_{slab} \approx 0.1$ [e.g., Matthaeus et al., 2003, and references therein] is interesting in that it yields a very high injection threshold in the neighborhood of $90^\circ$, and the $90^\circ$ value is still 4–5 orders of magnitude higher than at the corresponding parallel shock. However, the injection energies at 1 AU, by contrast, while still depending on $\lambda_{2D}/\lambda_{slab}$, do not exhibit the same degree of variability. Injection thresholds now range from $\sim 2$ keV (parallel and perpendicular shocks) to $\sim 40$ keV ($\theta_{inj} \sim 80^\circ$), depending on $\lambda_{2D}/\lambda_{slab}$. The differences in the injection threshold for the termination shock case compared to the 1 AU example...
are due to the difference in the fluctuating magnetic field energy density \( \langle b^2 \rangle / B^2 \). After peaking near 90°, \( T_{\text{inj}} \) plummets as \( \theta_{\text{bn}} \to 90° \). At 1 AU, rather remarkably, the lowest injection energies occur for shocks that are either parallel or almost strictly perpendicular regardless of the ratio \( \lambda_{\text{2D}} / \lambda_{\text{slab}} \). By contrast, highly oblique shocks with \( \theta_{\text{bn}} \geq 45° \) have a high injection threshold.

The results for the injection energy thresholds may have two important consequences. At the termination shock, for example, the shock obliquity will vary constantly in response to variable upstream solar wind conditions. This can be true at interplanetary shocks and bow shocks as well. Since injection is much easier at either a parallel or strictly perpendicular shock, it is therefore possible that a shock will inject particles into a diffusive shock acceleration mechanism very intermittently, unless some preacceleration mechanism can boost particle energies significantly. However, since the heliospheric termination shock is likely to be quasiperpendicular most of the time [e.g., Zank, 1999] and injection energies will exceed 100 keV, it is likely that injection is either highly intermittent or results from preacceleration mechanism such as multiply reflected ion acceleration/shock surfing [Zank et al., 1996b; Lee et al., 1996] or second-order Fermi acceleration [Chalov, 2000; Kallenbach et al., 2005b]. For interplanetary shocks, a supra-energetic particle population is often present in the form of flare accelerated particles [Mason, 2000; Desai et al., 2004]. Since interplanetary shocks within 1 AU have regions that can be highly oblique or quasiparallel, the injection character can vary across a CME-driven shock, for example (Figure 6). This will however depend sensitively on the assumed \( \lambda_{\text{2D}} / \lambda_{\text{slab}} \) ratio. At a highly oblique section of the shock front, with \( \lambda_{\text{2D}} / \lambda_{\text{slab}} \sim 0.1 \), injection will either not occur (at least thermal solar wind particles cannot be accelerated in the context of diffusion theory) or a pre-existing energetic particle population, such as flare particles, which already have energies exceeding \( \sim 20 \text{ keV} \), will be accelerated. Consequently, the Fermi accelerated particles must reflect this injection bias and SEP and ESP events that posses an impulsive-like composition must therefore have been accelerated at a highly oblique shock. If instead, \( \lambda_{\text{2D}} / \lambda_{\text{slab}} \sim 1 \), the injection threshold at an oblique shock is \( \sim 5–6 \text{ keV} \), which is sufficiently modest that thermal heating of the solar wind particles by the shock may be sufficient; this will not, however, lead to a compositional bias in gradual events.

4. Shock Acceleration

By adapting the approach developed by Zank et al. [2000] and Li et al. [2003], we have developed a model for particle acceleration at a perpendicular interplanetary shock.

Figure 5. Same as Figure 4 except that the parameters are now appropriate to an interplanetary shock located at 1 AU. Note that the \( \theta_{\text{bn}} = 0° \) curve is almost completely obscured by the \( \theta_{\text{bn}} = 90° \) curve.

Figure 6. Schematic of a CME-driven shock illustrating the variation in shock obliquity and the corresponding regions of high injection energy thresholds.
Just as at a quasiparallel shock, the usual stationary cosmic ray transport equation can be solved at a perpendicular shock, yielding the same spectral dependence on the shock compression ratio in both cases. The differences between the two cases are the scaling of the upstream exponential decay of particle intensity, the injection energy, and the maximum energy, since all of these depend on the spatial diffusion coefficient.

[20] By taking the following three steps, we can utilize our earlier approaches [Zank et al., 2000; Li et al., 2003] directly.

[21] 1. From equations (9) and (10), we can evaluate \( \kappa_\perp = 1/3v_0\lambda_\perp \) on assuming (1) a Parker spiral magnetic field [e.g., Jokipii and Kota, 1989].

\[
B = B_0(r_0/r)^2 \left( 1 + \left( \frac{\Omega_0r}{U_{sw}} \right)^2 \right)^{1/2},
\]

where \( \Omega_0 \) is the solar angular velocity, and \( B_0 \) the magnetic field at the heliocentric corotation radius \( r = r_0 \); (2) a decomposition of the upstream magnetic energy density in the turbulence according to

\[
\langle b^2 \rangle = \langle b_{slab}^2 \rangle + \langle b_{2D}^2 \rangle,
\]

in the proportion \( \langle b_{slab}^2 \rangle : \langle b_{2D}^2 \rangle = 20\%: 80\% \), and (3) that \( \langle b^2 \rangle \) decays according to a simple WKB model

\[
\frac{\langle b^2 \rangle}{\langle b_0^2 \rangle} = \left( \frac{r}{r_0} \right)^{-3},
\]

where \( \langle b_0^2 \rangle \) is the magnetic field variance at a reference heliocentric distance \( r_0 \). The WKB approximation is reasonable within 5–8 AU but beyond that, a more elaborate turbulence-based model is needed [Zank et al., 1996a]. These assumptions allow us to evaluate the perpendicular diffusion coefficient \( \kappa_\perp \) at the shock throughout the heliosphere. We note that we have implicitly assumed that no significant instabilities will be generated in the foreshock of a highly perpendicular shock. However, as discussed by Zank et al. [1990] and Zank et al. [2005], a cosmic ray mediated perpendicular shock can be unstable to magnetoanisotropic instabilities, which may have an effect on particle acceleration. This is not pursued here.

[22] 2. By setting \( \xi = 1 \) in equation (16) (although we should strictly use \( \xi \ll 1 \)), we can evaluate the local injection momentum or velocity if particles are to be accelerated diffusively at highly oblique shocks. Thus the injection velocity is given by

\[
v_{\text{inj}} = 3 \left[ \frac{g}{(s - 1)^2} + \left( \frac{\kappa_\perp + \kappa_\parallel \cos^2 \theta_{\text{sh}}}{\kappa_\parallel \sin^2 \theta_{\text{sh}} + \kappa_\perp \cos^2 \theta_{\text{sh}}} \right) \right]^{1/2},
\]

in general, or for nearly perpendicular shocks, by

\[
v_{\text{inj}} = 3 \left[ \frac{1}{(s - 1)^2} + \left( \frac{\lambda_\perp + \lambda_\parallel \cos^2 \theta_{\text{sh}}}{\lambda_\parallel \sin^2 \theta_{\text{sh}} + \lambda_\perp \cos^2 \theta_{\text{sh}}} \right) \right]^{1/2}.\]

Equation (18) or (19) represents a far more stringent injection condition than the corresponding condition used by Zank et al. [2000] or Li et al. [2003], which essentially uses downstream thermalization as a criterion for injection.

[23] 3. To evaluate the upper limit on the diffusively accelerated spectrum, we need to equate the shock acceleration timescale and the available time for acceleration, i.e., the shock dynamical timescale \( R/R_{\text{sh}} \), where \( R(t) \) describes the shock position at time \( t \). Thus

\[
\tau_{\text{acc}} = \left( \frac{1}{r} \right) \left( \frac{dp}{dt} \right) = \frac{3\sqrt{8}}{\gamma_{\text{sh}}(s - 1)} \Rightarrow t = \frac{3s}{R} = \frac{3\sqrt{8}}{\gamma_{\text{sh}}(s - 1)} \int_{p_{\text{min}}}^{p_{\text{max}}} \kappa_\perp(p) \frac{dp}{p} = \frac{3\sqrt{8}}{\gamma_{\text{sh}}(s - 1)} \int_{p_{\text{min}}}^{p_{\text{max}}} \kappa_\perp(p) \frac{dp}{p},
\]

where \( p \) is the momentum per nucleon. Conversely, for downstream thermalization, the charge

\[
\frac{\langle b^2 \rangle}{\langle b_0^2 \rangle} = \left( \frac{r}{r_0} \right)^{-3},
\]

we can then solve (20) numerically. For the purposes of illustration, it is simpler to assume that \( \kappa_\perp \approx \alpha \kappa_{\|} \) (equation (9), where \( \alpha \) is a constant and lies in the range \( \alpha = 10^{-1} - 10^{-2} \) [see Zank et al., 2004]). We need to consider two cases. In the inner heliosphere (within several AU of the sun), the most energetic particles resonate with the inertial range; that is, the maximum particle gyroradius is much less than the correlation length scale. By contrast, with the weakening of the magnetic field in the outer heliosphere, very energetic particles can resonate with the energy containing range, which implies that the particle gyroradius exceeds the correlation length. In the former case, \( r_L \ll \lambda_{\text{sh}} \), from which it follows that \( D \approx 0 \) in (10). We can then approximate the parallel diffusion coefficient for ions of arbitrary mass and charge as

\[
\kappa_{\|} \approx 1.046 \frac{B^{5/3}}{\langle p_{\|}^2 \rangle^{1/3} A^{1/3} (\lambda_{\text{sh}}/Q)^{1/3}},
\]

where \( m_p \) is proton mass, \( e \) the electron charge, \( Q \) the charge state, \( A \) the mass number, and we have used \( \lambda_{\text{sh}} = 1.34 \lambda_{\text{slab}} ^{1/3} \). Equating the dynamical timescale and the particle acceleration timescale in this limit yields

\[
\tilde{p}_{\text{max}} \approx \left( \frac{Q}{A} \right)^{1/4} \left( \frac{e}{m_p} \right)^{1/4} (\lambda_{\text{sh}})^{-1/2} \frac{\gamma_{\text{sh}}(s - 1) R}{1.048 \lambda_{\text{sh}}^{1/3} \langle b_{\text{slab}}^2 \rangle^{3/4}},
\]

where \( \tilde{p} \) is the momentum per nucleon. Conversely, for energetic particles with gyroradii that exceed the correlation length scale, it is easily seen that \( D \gg 1 \) in the quasilinear expression for \( \kappa_{\|} \) (equation (10)). In this limit, \( D \approx 6.02 \times 10^{-2} (p_{\|}/\lambda_{\text{sh}}^{1/3}) ^{1/2} \). For arbitrary mass and charge ions, we compute the maximum particle momentum as

\[
\tilde{p}_{\text{max}} \approx \left( \frac{Q}{A} \right)^{2/3} \left( \frac{e}{m_p} \right)^{2/3} \frac{R}{s} \frac{V_{\text{sh}}^2}{1.17 \alpha \lambda_{\text{sh}}^{1/3} \langle b_{\text{slab}}^2 \rangle}.
\]
The maximum momentum expressions reveal several interesting points. A fundamental difference between the perpendicular and quasiparallel expressions is that the former is derived from a quasilinear theory based on pre-existing turbulence in the solar wind, whereas the latter results from solving the coupled wave energy and cosmic ray streaming equation explicitly, i.e., in the perpendicular case, the energy density in slab turbulence corresponds to that in the ambient solar wind whereas in the case of quasiparallel shocks, it is determined instead by the self-consistent excitation of waves by the accelerated particles themselves. From another perspective, unlike the quasiparallel case, the resonance condition does not enter into the evaluation of $p_{\text{max}}$ given in (23). As a result, the diffusion coefficient is fundamentally different in each case, and hence the maximum attainable energy is different for a parallel or perpendicular shock. For a perpendicular termination shock, for example, there is no dependence on the strength of the background magnetic field $B$ (equation (23)). In this case, the primary dependence is on shock size/age ($R$) and shock speed ($R$ and $V_{\text{sh}}$) and the radial variation of the energy density in slab turbulence ($\tilde{h}^2_{\text{lab}}$). Conversely, in the inner heliosphere where the mean magnetic field is strong, the maximum momentum decreases with increasing field strength, this reflecting the increased “tension” in the mean field.

Some comments regarding the determination of $p_{\text{max}}$ for different shock configurations and ionic species are useful since three approaches have been identified in this and previous work [Zank et al., 2000; Li et al., 2003].

1. For protons accelerated at a quasiparallel shock, $p_{\text{max}}$ is determined purely on the basis of balancing the particle acceleration time resulting from resonant scattering with the dynamical timescale of the shock. The wave/turbulence spectrum excited by the streaming energized protons extends in wave number as far as the available dynamical time allows.

2. For heavy ions at a quasiparallel shock, the maximum energy is also computed on the basis of a resonance condition but only up to the minimum $k$ excited by the energetic streaming ions, which control the development of the wave spectrum. Thus maximum energies for heavy ions are controlled by the accelerated protons and their self-excited wave spectrum. This implies a $(Q/A)^2$ dependence of the maximum attainable particle energy for heavy ions.

3. For protons at a highly perpendicular shock, the maximum energy is independent of the resonance condition, depending only on the shock parameters and upstream turbulence levels. For heavy ions, this implies either a $(Q/A)^{1/2}$ or a $(Q/A)^{4/3}$ dependence of the maximum attainable particle energy, depending on the relationship of the maximum energy particle gyroradius compared to turbulence correlation length scale. The summary presented here suggest that it may be possible to extract observational signatures related to the mass-charge ratio that distinguish particle acceleration at quasiparallel and highly perpendicular shocks.

The steps described above allow us to immediately apply the approach of Zank et al. [2000]. In the following, we consider a CME-driven shock propagating from 0.1 AU to ~1 AU. For the purposes of illustration, we make the somewhat simplistic assumption that the shock remains either parallel or that $\theta_{bn} = 85^\circ$, and for the parallel shock, we use the downstream thermalization criterion of Zank et al. [2000]. We also assume that $\lambda_{2D}/\lambda_{\text{lab}} = 0.1$. The dependence of injection energy and the maximum energy to which a particle can be accelerated as a function of radial distance is illustrated in Figure 7 for a perpendicular shock.
In addition, the figure compares the corresponding energies for an otherwise identical parallel shock where the energetic particles excite the upstream wave spectrum (a subtlety that has been neglected in all previous comparisons of particle acceleration at perpendicular and parallel shocks \cite[e.g.,][]{jokipii1987}; that is, we solve (1)–(3) assuming that the shock remains parallel for the duration of its propagation to 1 AU. Since we assume wave excitation in the parallel shock calculation, this results in maximum energies that can be as much as an order of magnitude larger at a parallel shock than at a perpendicular shock close to the sun. The maximum energies accelerated at parallel and perpendicular shocks begin to converge towards 1 AU. The decay in maximum energy is slower for the perpendicular shock than the parallel because of the slower WKB-like decrease in energy in turbulent fluctuations and because of the $1/r^2$ dependence of $B$ within 1 AU of $p_{\text{max}}$ for the parallel case. We stress that in the absence of the self-consistent wave excitation, the perpendicular shock would accelerate particles to higher energies than a parallel shock. This suggests that the question of perpendicular versus parallel diffusive shock acceleration at the heliospheric termination shock \cite{jokipii1987} may need to be revisited. The corresponding evolution in the diffusion coefficients (perpendicular and parallel) at different heliocentric distances is illustrated in Figure 8.

Finally, since the injection energy at a quasiperpendicular shock is much higher than at the corresponding quasiparallel shock, we can expect signature differences in the composition of accelerated particles. We can utilize the interplanetary shock acceleration models of Zank et al. \cite{zank2000} and Li et al. \cite{li2003,li2005} for perpendicular shock acceleration to derive spectra, intensity profiles, etc. for SEP and ESP events.

In Figure 9, we show the evolution of the accelerated energetic particle relative number density at 1 AU as a function of time for the example of a quasiperpendicular and parallel shock. The solid curves correspond to the quasiperpendicular shock $\theta_{\text{inj}} = 85^\circ$ and the dotted curves to the parallel shock accelerated protons. The total number of injected particles in the parallel shock case is assumed to be 20 times greater than that of the perpendicular shock. This ratio is in agreement with results obtained by Mewaldt et al. \cite{mewaldt2001}, who considered energetic particle fluences over a 3 year period and found that the superthermal tail of the solar wind (above 0.01 MeV/nucleon) can be approximated by an $E^{-2}$ power law. By assuming that the seed particles have such a power law, and that the injection energies are as shown in Figure 7, allows us to estimate the injected particles number density at a perpendicular shock relative to that at a parallel shock. It is clear from Figure 7 that the injected number of particles at a perpendicular shock, compared to that at a parallel shock, increases with time as the shock propagates away from the sun. This is because the particle injection energy at a perpendicular shock decreases faster with heliocentric distance than that at a parallel shock (see Figure 7). The ratio of the injection energy $E_{\text{inj}}/E_{\text{inj}}^k$ is about 50 at 0.1 AU, 17 at 0.5 AU and 4 at 1 AU. This translates to an injected particle ratio of 0.02, 0.06 and 0.25, respectively. In this work, since our model is crude, we did not follow the time evolution for the injected particle at the perpendicular shock, but instead assumed that the total number of injected particles at the quasiperpendicular shock remains 20 times smaller than that at the parallel shock.

The comparative shapes of the spectra are interesting. At early times, the quasiperpendicular shock has a much clearer power law extending from lower energies than the parallel case. This is a consequence of quasiperpendicular shocks not being as effective at trapping particles at the shock front as parallel shocks, which ensures that relatively
more particles escape at lower energies from the highly perpendicular shock than at the parallel shock. Consequently, a power law spectrum is more likely to be seen at earlier times. The parallel shock can accelerate particles to higher energies and this is revealed clearly in Figure 9. The parallel shock spectra tend to fill out into a power law over time until at 1 AU it almost resembles the perpendicular shock example, except that it is shifted out to higher energies. In summary, Figure 9 reveals clear differences between particle spectra measured at 1 AU for parallel and quasiperpendicular shocks.

[33] Shown in Figure 10 are the corresponding intensity profiles for a quasiperpendicular (top panel) and parallel (bottom panel) shock. Three energies are illustrated. Clear differences between the two models are evident. Thanks to wave excitation by the streaming instability at the parallel shock, the parallel diffusion coefficient is smaller at these energies (Figure 8) than the diffusion coefficient at the quasiperpendicular shock. Consequently, particle trapping is more efficient at the parallel shock, thus limiting particle escape at the energies illustrated. This is reflected in the intensity profiles, which show a very rapid rise time and formation of a plateau in the quasiperpendicular shock case. A much slower rise time is exhibited in the parallel shock example and only the $T = 50$ MeV protons reach a plateau phase (by contrast, the 50 MeV particles accelerated at the quasiperpendicular shock are released rapidly enough that they are no longer observed at 1 AU after ~1 day). Like the spectra illustrated in Figure 9, distinctive differences are present in the intensity profiles associated with quasiperpendicular and parallel shocks. These will of course not be revealed as clearly in observations as they are in our models since we have deliberately neglected the changing obliquity of the shock with heliocentric radius and the spacecraft connection to different magnetic field lines with time. The compositional bias and the grouping of events identified by Tylka et al. [2005] obviously cannot be addressed within the context of this work since we have focused primarily on proton acceleration at perpendicular shocks. However, this work provides a framework in the same fashion that the Zank et al. [2000], Li et al. [2003] and Rice et al. [2003] work provided the framework for the heavy ion models developed by Li et al. [2005a]. The extension of this work to include heavy ions and the changing obliquity of the shock normal with increasing heliocentric distance will be pursued.

5. Observations

[34] If the model for particle acceleration at a perpendicular shock is correct, we should expect to see at least an energetic particle spectrum downstream of the shock and virtually no wave activity upstream of the shock. By contrast, a parallel shock should exhibit significant wave power enhancement upstream associated with the energetic particle population. We used ACE MAG and EPAM data to investigate particle acceleration at shocks in the inner heliosphere. In particular, we found a quasiparallel shock example (Figure 11), which might be regard as “typical” from a theoretical perspective, and contrast this with two perpendicular shock examples (Figure 12). We examine the magnetic field activity at three locations upstream of each of the shocks, measure the evolution of the particle spectrum at ACE as the shock approaches, and compute the evolution in
the spectral index of the accelerated particle spectrum. Such studies of the upstream wave activity remain comparatively rare with only a few cases published [Kennel et al., 1986; Sanderson et al., 1985; Kallenbach et al., 2005a; Bamert et al., 2004; Lario et al., 2005].

The power spectral density was obtained using the approach of Bieber et al. [1996], and we used 1-s cadence ACE/MAG [Smith et al., 1998] measurements. The shock parameters, including the speed in the upstream plasma frame, \(V\), the asymptotic compression ratios of density and magnetic field, \(s_n\) and \(s_b\), the Alfvén Mach number, \(M_A\), and \(q_{bn}\), were obtained by fitting the Rankine-Hugoniot relations [Szabo, 1994] to the ACE data. These values are printed on the figures.

The magnetic field strength \(B\) and the underlying rms value \(\langle B_{rms}\rangle\) measured by the ACE magnetometer at 1 AU during day 297 of year 2003. We examine the magnetic power spectral density (PSD) at three marked intervals, one well ahead of the shock (blue vertical lines), somewhat ahead (green lines), and immediately upstream of the shock (red lines). The PSDs for the intervals are plotted in the second panel down in the corresponding colors. As the shock approaches the spacecraft, the energy in fluctuations increases across the measured spectral range (10\(^{-4}\) – 0.5 Hz). There is also a noticeable departure from a power law above 10\(^{-2}\) Hz in the PSD immediately upstream of the shock [see also Lario et al., 2005]. This feature was identified already by Kallenbach et al. [2005a] and Bamert et al. [2004], who ascribed it to the excitation of magnetic fluctuations by the streaming instability as protons escape upstream. Energetic ion intensities measured by the LEMS120 telescope of the EPAM instrument on ACE [Gold et al., 1998] shown in the third panel of Figure 11 display the time-intensity profiles typical of diffusive shock-acceleration in the presence of resonant waves with a slow rise at low energies as the shock approaches the observer (see also Figure 1). Higher energies show a temporal decline rather than rise in the intensity profile, for reasons discussed by Zank et al. [2000] and Li et al. [2003].

Vertical lines on the intensity plots identify the time at which energetic particle spectra, plotted in the fourth panel down of Figure 11, were observed at 1 AU. These spectra therefore correspond to the theoretical spectra shown in Figure 9. The energy plotted ranges from 42 to 321 keV. Just as illustrated in the quasiparallel model show in Figure 9, the spectrum “unfolds” at lower energies,
converging to a spectrum that is almost unchanging immediately upstream and downstream of the shock. The spectrum at higher energies is close to a power law and hardens at lower energies. Such behavior is consistent with theoretical models of particle acceleration at a time-dependent evolving interplanetary shock [Zank et al., 2000; Li et al., 2003]. The bottom panel of Figure 11 plots the spectral index \( \gamma \) as a function of time obtained by fitting an \( E^{-\gamma} \) power law to the spectrum observed at 1 AU between the energies 47 keV to 4.8 MeV. This plot illustrates explicitly the unfolding of the low energy spectrum with time.

[38] We have identified several cases of highly perpendicular shocks which exhibit both evident particle acceleration and an absence of associated wave/turbulence activity. Two examples are plotted in Figure 12, in the same format as Figure 11. The left panels of Figure 12 correspond to a comparatively weak shock (compression ratio 2.8 ± 0.2, \( M_A = 3.6 \)) whereas the right panels correspond to a strong shock (compression ratio 3.7 ± 0.8, \( M_A = 5.9 \)). In Figure 12, the top two panels of both columns are clearly very different from those of the oblique case of Figure 11, with absolutely no enhancement in the magnetic PSD even immediately upstream of the shock. (Cane and Richardson [2003] identified a short time interval with solar wind characteristics typical of an ICME from 42/1600 UT to 42/2000 UT (i.e., from 42.666 to 42.8333), but it is difficult to see what influence it may have had on the upstream medium and especially on the possible absence of waves.) Referring to panels 3 and 4 of Figure 12, we see quite typical intensity profiles although somewhat more irregular and “spikey” than the oblique example. The shock on day 42 of 2000 shows a spike at the time of the shock passage that is more pronounced at high-energies. The shock passage was at 42/2319 UT (42.9646). Less pronounced peaks are present for the shock of day 118 of 2001. The energetic particle spectra are different for the weak and strong shock examples, with more “unfolding” occurring in the strong shock case whereas the weak shock spectra are relatively invariant with a strong “roll-over.” The observed evolution of the strong shock spectra is very similar to that of our calculated quasiperpendicular example (Figure 9) with spectral unfolding at low energies and the hard power law spectrum at high energies.

6. Conclusions

[39] Particle acceleration at a quasiperpendicular shock has long been an outstanding problem, in large part due to our poor understanding of the perpendicular component of

**Figure 11.** From top to bottom (panels 1–5), we plot the magnetic field (panel 1) upstream and downstream of a shock showing three intervals upstream for which the magnetic power spectral density was computed (2), (3) the energetic ion fluxes at various energy intervals using 1-minute spin-averaged intensities as measured by the LEMS120 telescope of the EPAM instrument on board ACE, (4) energy spectra measured at times indicated by the vertical lines in the intensity panel, and (bottom, 5) the evolution of the spectral index gamma obtained by fitting a power-law \( E^{-\gamma} \) to the intensities measured in the four lowest energy channels (47-321 keV). The time of the shock passage is indicated by the gray vertical line. The shock parameters are given in the panel 4. This figure corresponds to an oblique/quasiparallel shock.
the diffusion tensor. A key ingredient for particle acceleration at oblique shocks is the self-consistent excitation of the upstream scattering wave/turbulence field by the anisotropic, energetic particle distribution escaping upstream from an interplanetary shock. For highly oblique shocks, wave excitation is quenched and particle scattering has to therefore rely on in situ solar wind turbulence that is convected into the shock. We have developed a basic theory for particle acceleration at highly perpendicular shocks based on the convection of in situ solar wind turbulence into the shock, and the spatial perpendicular diffusion coefficient is determined on the basis of NLGC theory. This approach allows us to extend the dynamical particle acceleration and SEP models of Zank et al. [2000], Li et al. [2003], and Rice
et al. [2003] to include highly perpendicular shocks that cannot excite scattering waves. From this, we may address the questions of particle injection, maximum energies, intensity profiles, and the evolution of accelerated proton spectra throughout the heliosphere.

40. We may enumerate our primary results as follows.

41. 1. The NLGC model shows that the perpendicular mfp $\lambda_{90}$ is related simply to the parallel mfp $\lambda_{l}$ in the energy range $10^{-3}$–100 MeV with $\lambda_{90}/\lambda_{l} \approx 10^{-2}$ at low energies and $\sim 10^{-4}$ at 100 MeV for the termination shock example. Similar results hold for interplanetary shocks at 1 AU.

42. 2. Within the context of the NLGC theory, we find that the perpendicular diffusion coefficient depends rather sensitively on the ratio of correlation lengths $\lambda_{2D}/\lambda_{slab}$.

43. 3. The perpendicular acceleration timescale at high energies is significantly smaller than the Bohm acceleration timescale, which implies rapid acceleration at perpendicular shocks.

44. 4. The perpendicular mfp at 100 AU is not significantly different from that at 1 AU, especially at high energies. One implication is that the perpendicular acceleration timescale for perpendicular shocks is similar throughout the heliosphere. By contrast, 3 orders of magnitude separate the parallel mfp at 100 AU and 1 AU.

45. 5. For the diffusion approximation to be valid at a shock, the energetic particle anisotropy must be small. We find that the anisotropy is very sensitive (and singular) in the limit $\theta_{an} \rightarrow 90^\circ$. The particle anisotropy typically increases as $\theta_{an}$ increases from 0°, peaking at some $\theta_{an} < 90^\circ$ and then decreasing as $\theta_{an} \rightarrow 90^\circ$. Assuming that particles can be injected and accelerated diffusively at shock when the anisotropy is small allows us to show that the injection energy increases with increasing obliquity, reaching a maximum at some $\theta_{an} < 90^\circ$, before decreasing as $\theta_{an} \rightarrow 90^\circ$. Rather surprisingly, at 1 AU we find that the injection energy at a parallel shock and a highly perpendicular shock are the same! By contrast, highly oblique shocks require much higher injection energies than a parallel shock. Because of the decay of turbulence with increasing heliocentric distance, the injection threshold at the perpendicular termination shock can be similar or much higher than at the parallel termination shock, depending on the ratio of 2-D to slab correlation lengths.

46. 6. Two important implications follow from our results for injection at highly perpendicular shocks. (1) Injection at the termination shock is either likely to be highly intermittent (occurring only at locations that are highly parallel or perpendicular) or to require a pre-acceleration mechanism. (2) Highly perpendicular interplanetary shocks are more likely to accelerate a preexisting energetic particle population (possibly flare accelerated particles) than in situ solar wind particles, with obvious implications for compositional bias.

47. 7. The theoretical model for diffusive shock acceleration at perpendicular shocks can be applied to the SEP models developed by Zank et al. [2000] and Li et al. [2003]. For perpendicular shocks, the maximum energy to which particles can be accelerated is determined by the in situ upstream turbulence, unlike the case for parallel shocks where $p_{\text{max}}$ is determined by the resonance condition. This distinction between the two cases has important implications for the $Q/M$ scaling of $p_{\text{max}}$ at perpendicular and parallel shocks, which may be distinguishable observationally.

48. 8. With the inclusion of wave excitation at parallel shocks, the acceleration timescale for parallel shocks can be faster than at a perpendicular shock, a subtlety that has not been appreciated before. Consequently, we find that parallel interplanetary shocks can accelerate protons to higher energies than comparable perpendicular interplanetary shocks. The maximum energy to which a parallel shock can accelerate protons falls fairly rapidly with increasing heliocentric distance whereas it remains almost constant for highly perpendicular shocks.

49. 9. Subject to the somewhat simplistic but revealing assumption that a shock remains either parallel or highly perpendicular as it propagates from the Sun to 1 AU, we find that the evolving spectra and intensity profiles observed at 1 AU can be quite different for the two cases. The parallel shock spectra extend to higher energies but the highly perpendicular shock accelerated spectra are more power law-like than the more exponential parallel shock spectra. Easier injection at the parallel shock ensures that corresponding spectra have a greater relative number density than those corresponding to highly perpendicular shocks. The absence of a self-excited wave field at highly perpendicular shocks implies more efficient particle trapping at the parallel shock. Consequently, rise times for intensity profiles are much more abrupt for particles accelerated at a highly perpendicular shock than for quasiparallel shocks. Thus a plateau is established much more rapidly for highly perpendicular shocks, and, for similar reasons, intensity profiles can decay more rapidly for high energy particles, in marked contrast to the parallel shock case.

50. 10. We have identified several examples of highly perpendicular shocks that show no evidence of enhanced wave activity or turbulence upstream of the shock yet accelerate particles that have spectra and intensity profiles that are very reminiscent of diffusive shock acceleration. The highly perpendicular shock examples are compared to a “typical” oblique shock in which the wave excitation ahead of the shock is very evident.

Acknowledgments. G. P. Z., L. V., and Q. H. acknowledge the partial support of NASA grants NNG04GF83G, NNG05GH35G, and NNG05GM62G; a Cluster University of Delaware subcontract BART372159/8G; and NSF grants ATM0317509 and ATM0428880. D. L. acknowledges the support of NASA grant NAG5-13487, and C. W. S. acknowledges the support of an ACE Caltech subcontract 44A-1062037 and a GSFC subcontract NNG04GM05G. We thank the ACE EPAM and ACE MAG instrument teams and the ACE Science Center for providing the ACE data.

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